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LABOR ADJUSTMENT TO IMPORTS UNDER RATIONAL EXPECTATIONS

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The effects of imports on industry employment are often determined through the use of input-output studies. Input-output assumes that imports cause proportional and immediate effects on industry employment. Increases in imports will therefore be expected to cause large, sudden decreases in employment. The problem arises, however, that actual events are often poorly predicted by the input-output model.

20. To better predict the effects of imports on employment, a model of the demand for labor was developed that allowed for gradual adjustment in employment to perceived changes in output, where these changes arise either from cyclical factors or an increase in competing imports. What is expected to be produced in the future was felt to be an important determinant of current employment needs and therefore was explicitly included in the labor demand model.

According to our findings, expectations of future output are important determinants of industry employment demand in the majority of industries studied. Perhaps, more surprisingly, imports induce a slower adjustment in employment than does an equivalent change in GNP, the measure used to represent cyclical factors. Our results suggest input-output studies overestimate the effects of competing imports on employment in the industry.

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INTRODUCTION

Empirical studies of labor demand characteristically come up
with two results that conflict with predictions drawn from
simple static theory. First, theory suggests that, because
labor is usually assumed to be the most variable factor in
the short run, increasing amounts of it will be needed for
added increments of output. In fact, over the business
cycle, the opposite occurs. As output rises, employment in-
creases less than proportionately. Second, theory suggests
that, over the long run, with all inputs variable, returns to
scale will be constant or diminishing.[1] In fact, empirical
studies of labor demand appear to imply long-run increasing
returns to scale.

A widely accepted explanation for the implied increasing
returns is labor hoarding; during a downturn in output, firms
hold unneeded skilled workers "in inventory" in order to
avoid the cost of finding replacements when demand picks up
again. In accounting for labor hoarding, the first step has
been to use partial adjustment models. However, the ac-
counting has been incomplete--the implied returns to scale
are still increasing. We believe that this is, in part, a
consequence of failure to adequately model expectations.

A standard assumption, implicit in partial adjustment models
(including models of interrelated factor demand) is that
future values of the exogeneous variables are expected to be

---

[1]For a discussion of constant returns as a truism, see
Friedman [4], pp. 136-138. Engineering cost curves at the
plant level do tend to find only modest deviations from
constant returns to scale (see Scherer [19], pp. 94-98).
studies of labor demand appear to imply long-run increasing
returns to scale.

the same as the present values, or to differ by a time trend. This assumption, however, can not be appropriate as a description of how expectations move over the business cycle; employers have information from previous cycles which tells them that their business will recover from a recession.

Expectations of exogenous variables can be important if there are high fixed costs to changing employment levels. This importance has been noted by a number of economists including Gould [3], Brechling [1], and Nadiri and Rosen [14]. With the recent work on modeling expectations, especially rational expectations, labor demand models have begun to explicitly include the future (see Sims [20], Sargent [16], and Kennan [7]).

The Sims and Kennan papers both discuss the tendency to find increasing returns to labor in empirical studies. Sims attributes the findings to measurement error and the assumption of static expectations. By taking these into account, he does find a proportional response of employment to output, but only when the labor input is measured as man-hours, not when it is measured as the number of workers employed. Kennan finds sharply decreasing returns for both durables and nondurables, the latter to an extent which he considers unreasonable.

The usefulness of extending the study of the role of expectations in labor demand becomes clear in light of various government policies on employment adjustment. For example, it has been standard practice to use input-output studies to analyze the effect of imports on the domestic demand for the output of competing and related industries. In input-output

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analyses, it is assumed that imports cause proportional and immediate effects on industry employment. Rising imports, according to the input-output model, will cause large, sudden, decreases in employment.

The assumption of immediate and proportional adjustment does not accord with empirical evidence of gradual adjustment as well as labor-output elasticities less than one. The assumption would therefore require that imports have a special effect. A possible justification for a "special" effect of imports is that firms, upon seeing competing imports enter their market, interpret the change as permanent, completely revising their view of the future and adjusting employment accordingly, even if skilled workers (i.e., workers with high hiring and training costs) are involved. We test for a special effect of imports in a "rational expectations" model. The model must include two parts, one describing how expected output affects labor demand, the other describing how output (and, hence, expected output) is generated as a function of imports and other determinants.

Our procedure illustrates a useful characteristic of the theory of rational expectations: In the process of generating expectations the researcher automatically finds out how the different determinants in the model (e.g., imports) affect expectations. Alternative assumptions about the formation of expectations, such as surveys of anticipations or adaptive expectations, do not have this characteristic.

The model is applied to eleven industries at the two-digit SIC level. This disaggregation is important for theoretical reasons since industries differ in the amount and specificity

of human capital. As part of our study we: (1) disentangle
the effects of expectations from the process that generates
them; (2) make a direct comparison with a model of static
expectations; (3) compare the short-run effects on labor
arising from changes in imports and in GNP.

THE DEMAND FOR EMPLOYMENT[1]

In the theory of labor demand, a firm balances two motivations on its current holdings of labor: first, minimizing the cost of producing current output; second, keeping on hand enough workers to avoid large costs of adjusting to expected changes in output.

To derive a labor demand equation that incorporates those considerations, we begin with the assumption that, subject to a given production function, the employer minimizes the cost of producing a given stream of present and (expected) future output, including costs of adjustment.

To focus on the input of primary interest (the number of workers [N]), we combine all other inputs into a composite factor (Z). This includes inputs such as capital, capital utilization, and utilization of the labor force, e.g., average weekly hours or other unobservable measures of labor utilization.

The production function is of the general form:[2]

$$Q_t = f\left(N_t, Z_t\right), \quad f_N, f_Z > 0; \quad f_{NN}, f_{ZZ} < 0 . \tag{1}$$

---

[1]More complete derivations of the demand for employment are presented in appendices A and B, respectively. The first describes more carefully the approach taken in this section (see Sargent [16] and Levy [8]). The second is a different and perhaps more intuitive approach (see Levy [8]).

[2]A time trend could be included to represent other influences on labor demand that change smoothly over time, such as technological progress or the firm's capital stock. Since the derivation of labor demand is virtually the same, it is not included until the regression results are presented.

We assume this can be rewritten in inverse form:

$$Z_t = g\left(Q_t, N_t\right) \qquad (2)$$

and work with the second order Taylor expansion

$$Z_t = a + bN_t + \frac{c}{2} N_t^2 + dN_t Q_t + eQ_t + \frac{f}{2} Q_t^2 \qquad (3)$$

The (external) adjustment cost is also assumed to be quadratic.

$$\phi_t = \frac{\phi}{2} \left(N_{t+1} - N_t\right)^2, \quad \phi_N > 0, \quad \phi_{NN} > 0, \qquad (4)$$

where ϕ is a dollar cost of adjustment per man.[1] The firm's cost in period $t+j$ is:

$$
\begin{aligned}
C_{t+j} &= W_{t+j} N_{t+j} + q_{t+j} Z_{t+j} + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j}\right)^2 \\
&= W_{t+j} N_{t+j} + q_{t+j}\left(a + bN_{t+j} + \frac{c}{2} N_{t+j}^2 \right. \qquad (5) \\
&\qquad \left. + dN_{t+j} Q_{t+j} + eQ_{t+j} + fQ_{t+j}^2\right) \\
&\qquad + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j}\right)^2
\end{aligned}
$$

where W_{t+j} = wage paid to labor in time period $t+j$

$\qquad\;\; q_{t+j}$ = wage paid to input Z in time period $t+j$

The demand for labor at time $t+1$ will emerge as the solution to the minimization of the discounted expected flow of costs to the firm (V_t). Substituting for Z in (5) from (3), the required present value is:

---

[1] This cost function displays increasing marginal costs and so is consistent with lagged adjustment of N.

$$V_t = E_t \sum_{j=0}^{\infty} (1+r)^{-j} \left[W_{t+j} N_{t+j} + q_{t+j} \left(a + b N_{t+j} \right. \right.$$

$$\left. + \frac{c}{2} N_{t+j}^2 + d N_{t+j} Q_{t+j} + c Q_{t+j} + \frac{f}{2} Q_{t+j}^2 \right) \qquad (6)$$

$$\left. + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j} \right)^2 \right]^1$$

We assume that wages $\left(W_t \right)$, the price of input Z $\left(q_t \right)$, and adjustment costs $\left(\phi_t \right)$ increase by the same percent over time.[2] Thus, the firm faces only an exogenous process $\left\{ Q_{t+j} \right\}_{j=0}^{\infty}$ in its minimization problem.

Differentiating with respect to N_{t+j} yields the first order conditions:

$$E_{t+j} N_{t+j+1} - \frac{1}{h} N_{t+j} + a N_{t+j+1} = \frac{1}{\phi} X_{t+j} \qquad (7)$$

$$j = 0, 1, 2, \ldots$$

where $h \quad = \dfrac{1}{2+r+\frac{cq}{\phi}}$

$\qquad a \quad = 1 + r$

$\qquad X_{t+j} = W + bq + dq Q_{t+j}$

Equation (7) describes an infinite sequence of equations. Their solution is facilitated by rewriting (7) as

$$\left(1 - \frac{1}{h} L + a L^2 \right) E_{t+j} N_{t+j+1} = \frac{1}{\phi} X_{t+j} \qquad (8)$$

where L is the lag operator $\left(L y_t = y_{t-1} \right)$.

---

[1] E_t is the expectation operator. $E_t Y_{t+j}$ is the expected value of Y_{t+j} based on information in the $t\underline{\text{th}}$ period.

[2] A generalization of the model that allows for different growth paths for the different prices is presented later in the paper.

Factoring the lag polynomial yields:

$$\left(1-\lambda_1 L\right)\left(1-\lambda_2 L\right) E_{t+j} N_{t+j+1} = \frac{1}{\phi} X_{t+j} \qquad (9)$$

where $\lambda_1 + \lambda_2 = \frac{1}{h} = a + \frac{\phi + cq}{\phi}$ $\qquad (10)$

and $\lambda_1 \lambda_2 = a^1$

It can be shown that for any finite ϕ, $\lambda_1 < 1$ and $\lambda_2 > a > 1$.
Using the forward inverse of $\left(1-\lambda_2 L\right)$, since $\lambda_2 > 1$, we obtain

$$\left(1-\lambda_1 L\right) E_{t+j} N_{t+j+1} = -\frac{\left(\phi \lambda_2\right)^{-1} L^{-1}}{1-\lambda_2^{-1} L^{-1}} X_{t+j} \qquad (11)$$

$$= -\frac{\lambda_1}{a\phi} \frac{1}{1-\lambda_2^{-1} L^{-1}} E_{t+j} X_{t+j+1}$$

$$\left(\text{since } L^{-1} X_{t+j} = X_{t+j+1}\right)$$

Writing the denominator as

$$\left(1-\lambda_2^{-1} L^{-1}\right) = 1 + \lambda_2^{-1} L^{-1} + \lambda_2^{-2} L^{-2} + \ldots \qquad (12)$$

and using this expression in equation (11) leads to the final
equation for labor demand in the t+j+1st period:

---

[1]A solution for λ_1 may be determined from equation 10. It
turns out that

$$\lambda_1 = \frac{2ha}{1+\sqrt{1-4h^2 a}}$$

This can be shown to be less than 1 for any bounded and
positive ϕ and greater than 0 for any positive r.

$$N_{t+j+1} = \lambda_1 N_{t+j} - \frac{\lambda_1}{a\phi} \sum_{i=0}^{\infty} \left(\frac{\lambda_1}{a}\right)^i E_{t+j+1} X_{t+j+1+i} \qquad (13)[1]$$

[1]This minimization and its solution is a special case of minimizing over a quadratic objective function with an infinite horizon. The general problem is discussed in papers by Simon [18], Theil [21], and Sargent [17], specific models concerned with labor demand in Kennan [7] and Sargent [16].

THE MODEL FOR GENERATING EXPECTATIONS

Equation (13) indicates that labor demand depends on current output and future outputs in a declining geometric pattern. Employers do not know future output, and so must act on the basis of expectations. We assume that output is generated by the following model.[1]

$$\ln D_t = \alpha_0 + \alpha_1 \ln Y_t + \alpha_2 \ln\left(Y_t/Y_{t-1}\right) + \alpha_3 \ln P_t \qquad (14)$$
$$+ \alpha_4 D1 + \alpha_5 D2 + \alpha_6 D3 + \alpha_7 t$$

$$\ln Y_t = \beta_0 + \beta_1 \ln Y_{t-1} + \beta_2 \ln Y_{t-2} + \beta_3 D1 + \beta_4 D2 \qquad (15)$$
$$+ \beta_5 D3 + \beta_6 t$$

$$\ln M_t = \gamma_0 + \gamma_1 \ln M_{t-1} + \gamma_2 \ln M_{t-2} + \gamma_2 D1 \qquad (16)$$
$$+ \gamma_4 D2 + \gamma_5 D3 + \gamma_6 t$$

$$\ln P_t = \delta_0 + \delta_1 \ln P_{t-1} + \delta_2 \ln P_{t-2} + \delta_3 D1 \qquad (17)$$
$$+ \delta_4 D2 + \delta_5 D3 + \delta_6 t$$

$$Q = D-M \qquad (18)$$

where   Q   is domestic production

D   is total demand for an industry's products (includes both domestic production and imports)

M   is imports

Y   is constant dollar GNP

P   is the wholesale price index for the industry's output; relative to the overall wholesale price index

---

[1] These equations represent the basic version of the model. To capture differences among industries the actual regression equations will include only significant terms and some include alternative specifications of key variables e.g., to capture cyclical elements, the variable $\ln\left(Y_t/Y_{t-4}\right)$ may be used instead of $\ln\left(Y_t/Y_{t-1}\right)$.

D1,D2,D3 are dummy variables used to account for seasonal
        factors
    t   is a time trend
   AP   is the average value of P over the current and
        three preceding periods

All variables except the dummy variables, the time trend, and
GNP are specific to the individual industries.[1]

To summarize, for a specific industry, total demand (= do-
mestic output plus imports) is expressed as a function of
variables such as real GNP, relative prices (WPI of the
industry/WPI of all manufactured goods), time, and seasonal
dummy variables. Domestic output is determined as the
difference between total demand and imports, the latter
treated as exogenous.

The essence of rational expectations is that expectations are
made according to the same statistical process that generates
the actual variable. Hence, the model above is also a model
of expectations. The model can be used to form expectations
one period forward given the current and lagged information.
For example, imports one period forward are projected from
equation (16) with M_{t-1} now referring to the current period
and M_{t-2} now referring to last period. Imports two periods
forward are estimated with the same equation, with M_{t-2} re-
ferring to the current period and M_{t-1} referring to the fore-
case one period forward. To obtain expectations of output,
the same recursive forecasting scheme is applied to the model

---

[1]Note that the equation for imports (16) does not include the
price of imports relative to domestic; this is a consequence
of a lack of data on import prices at the 2-digit level.

as a whole. In other words, rational forecasts several
periods forward are formed by making use of nearer term
rational forecasts.[1]

---

[1]The statistical theory behind this technique is discussed in
an appendix available on request (or in [8]). Malinvaud [18]
and Sargent [17] discuss these issues as well.

DATA

To estimate the equations for labor demand and for generating expectations, we used quarterly data at the two-digit level on imports, output, prices, and employment in the following industries:

- Textile Mill Products (SIC 22)

- Apparel and Other Textile Products (23)

- Paper and Allied Products (26)

- Rubber and Plastic Products (30)

- Leather and Leather Products (31)

- Stone, Clay, and Glass Products (32)

- Primary Metals (33)

- Fabricated Metal Products (34)

- Machinery, Except Electrical (35)

- Electrical Equipment and Supplies (36)

- Transportation Equipment (37)

These industries exhibited varying degrees of import penetration--from less than 3 percent to almost 23 percent.

Empirical estimation of the relationship between domestic output and imports by industry requires data on prices of

domestic and imported goods. Unfortunately, these data are
unavailable at the two-digit level. This is even true for
unit values, the type of import prices typically used for
very highly aggregated or highly disaggregated products. At
the two-digit level, the best available measure was the
quarterly value of imports by industry from the first quarter
of 1968 to the fourth quarter of 1977. These data were
collected by the Bureau of Labor Statistics.

To use value data without the individual deflators, it was
therefore necessary to make an important assumption--namely,
that domestic products are perfect substitutes for products
of the same kind supplied from foreign firms. This commonly
used assumption implies that elasticities of substitution
between these products are infinite and that the correspond-
ing price ratios are constant. It means that the wholesale
price index of the domestically produced good is an appro-
priate deflator (at least, up to a constant) for the imported
good as well.

Total demand for a product (total demand = domestic output
plus imports) is specified as a function of variables in-
cluding real GNP, relative prices (WPI of the industry/WPI of
all manufactured goods), time, and seasonal dummy variables.
Imports are determined exogenously so that domestic output is
determined as the difference between total demand and
imports. The same variables that are in the total demand
equation would be in the domestic output equation except that
domestic output is also dependent upon imports.

Although the assumptions made above are not as general as one
might like, they are probably the best alternative when a
proper measure of import prices can not be found. At worst,
they probably do no more than overstate the effect of imports

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on domestic output, thereby putting an upper bound on the decrease in labor demand due to imported products.

Since total demand is defined to be equal to domestic output plus imports, it was necessary to use a value figure for domestic output. The measure used was taken from shipments and inventory data collected by industry. For nine of the industries (all except 23 and 31), total demand was calculated as the sum of shipments, the change in inventories, and imports. Shipments plus the change in inventories is a measure of industry production and is therefore an important determinant of labor demand. Since shipment and inventory data are measured in dollar value, the units match the import data. This would not have been true for many other measures, most notably the Federal Reserve Board Index of Production. Real values for both total demand and imports were obtained by deflating by the wholesale price index for each two-digit industry.

The shipment and inventory data were obtained from the Bureau of the Census publication, Manufacturers' Shipments, Inventories and Orders 1955-1977 [26] and all wholesale price indexes, including a general one for all manufactured goods, were obtained in the BLS publication, Producer Prices and Price Indexes [29].

For industries 23 and 31, only annual shipment data were obtainable from the above source. Quarterly indexes of production (from the FRB) were used to convert the annual figures to quarterly data. Thus, real total demand in these industries was measured as the sum of shipments and imports divided by the corresponding wholesale price index.

Finally, the measure of employment used was the number of
production workers in each industry. These data were
obtained from the Bureau of Labor Statistics publication
Employment and Earnings [27], the standard source for
employment data by industry.

EMPIRICAL RESULTS

Empirical estimation of the model proceeded in two parts.
First, for each industry, the three-equation system used to
generate expectations ((14), (16), (17)) was estimated using
OLS or, when appropriate, a GLS correction for serial cor-
relation. The regression equations are presented in appendix
C. The estimated equations were then solved to generate
forecasts of output. Second, labor demand (13) was estimated
using nonlinear least squares with the infinite distributed
lead in expected output truncated at eight quarters.[1,2] The
nonlinear estimates of labor demand are shown in table 1.

For the hypothesis that expectations are important (and
generated as assumed here), the crucial coefficient is b_3
which from equation (13) is equal to λ_1/a. A high value for
b_3 implies a strong effect of future output on current labor
demand. Estimates are positive in all industries and signif-
icant in seven of the eleven. The industries can be grouped
by b_3 as shown in table 2.

---

[1]The use of eight quarters, or two years, reflects the view
that this time frame adequately captures the firm's planning
horizon. Although it is true that tests over many different
horizons might lead to slightly different results, the number
of industries studied limits experimentation. Experimenta-
tion with longer leads in a few industries yielded similar
results.

[2]Conceptually, the truncation of the expectation series at
eight leads in the future means that the last coefficient has
a somewhat different interpretation. The truncation implies
that $Q^*_{t+9} = Q^*_{t+10} = \ldots$, so that the coefficient on the last
expectation series used $\left(Q^*_{t+9}\right)$ is really $b_3^9/\left(1-b_3\right)$. This
number will vary from b_3^9 by a negligible amount and so is
ignored in the computation of the estimated elasticities
presented in this section.

TABLE 1

LABOR DEMAND REGRESSION EQUATIONS BY INDUSTRY
(Current and Expected Output)

$$N_{t+1} = b_1 + b_2 \sum_{i=1}^{9} b_3^i Q_{t+i}^* + b_4 b_3 N_t + b_5 \text{ TIME} + b_6 D1 + b_7 D2$$

$$+ b_8 D3 + b_9 D37^*$$

Coef-ficient	Industry					
	22	23	26	30	31	32
b_1	9.24 (.112)	75.1 (.790)	112. (1.80)	210. (4.32)	9.51 (.261)	86.7 (3.09)
b_2	.937 (.815)	12.2 (.925)	1.13 1.07)	65.5 (.365)	22.2 (.137)	10.4 (1.37)
b_3	.791 (3.16)	.356 (1.57)	.703 (3.38)	.093 (.466)	.142 (.153)	.333 (2.48)
b_4	1.00 (4.03)	1.88 (1.77)	.784 (4.12)	.217 (.149)	5.68 (.156)	1.30 (3.57)
b_5	-.729 (-2.79)	.491 (1.29)	-1.42 (-3.97)	-1.58 (-4.14)	.237 (.697)	-.793 (-5.96)
b_6	-2.36 (-.370)	-43.1 (-5.46)	-4.71 (-1.22)	-10.7 (-1.58)	-4.73 (-1.59)	15.3 (4.91)
b_7	9.85 (1.58)	-16.3 (-1.62)	7.75 (1.88)	-16.8 (-1.55)	2.34 (.673)	27.8 (5.43)
b_8	11.3 (1.76)	-29.1 (-3.66)	4.96 (1.38)	14.7 (2.28)	-2.56 (-1.05)	18.9 (6.18)
b_9	--	--	--	--	--	--
R^2	.874	.940	.879	.894	.977	.956
DW	1.16	1.421	1.091	2.01	1.12	1.28

For industries 22, 26, 30, 32, 34, 35, 36, the range of the
regressions was Q2 1968 to Q4 1977. For industries 23 and 37,
the range was Q3 1968 to Q4 1977. For industry 33, the range
was Q4 1968 to Q4 1977.

*Dummy variable used to represent the major automobile strike
in Q4 1970.

TABLE 1 (Cont'd)

$$N_{t+1} = b_1 + b_2 \sum_{i=1}^{9} b_3^i Q_{t+i}^* + b_4 b_3 N_t + b_5 \text{ TIME} + b_6 D1 + b_7 D2 + b_8 D3 + b_9 D37*$$

Coef-ficient	Industry				
	33	34	35	36	37
b_1	23.3 (2.06)	-168.3 (-.603)	45.8 (.643)	20.8 (2.32)	452.8 (3.00)
b_2	4.63 (1.49)	1.43 (1.13)	.522 (.687)	4.54 (1.07)	3.78 (.588)
b_3	.454 (2.52)	.831 (3.30)	.820 (3.06)	.484 (2.50)	.296 (.915)
b_4	.822 (2.93)	.717 (6.42)	.961 (4.18)	1.04 (3.79)	1.65 (1.04)
b_5	-2.85 (-5.55)	-.083 (-.392)	-1.08 (-2.45)	-4.43 (-5.20)	-4.54 (-2.89)
b_6	-4.14 (-.412)	-7.21 (-.944)	-11.2 (-1.05)	5.60 (.450)	-17.8 (-.930)
b_7	5.07 (.438)	6.08 (.843)	-22.6 (-2.06)	14.5 (1.15)	-18.0 (-.773)
b_8	14.0 (1.48)	26.3 (4.22)	-7.74 (-.676)	48.9 (3.75)	53.2 (2.55)
b_9	--	--	--	--	-165.0 (-3.68)
R^2	.934	.951	.951	.926	.864
DW	1.38	1.90	.934	2.08	2.01

For industries 22, 26, 30, 32, 34, 35, 36, the range of the regressions was Q2 1968 to Q4 1977. For industries 23 and 37, the range was Q3 1968 to Q4 1977. For industry 33, the range was Q4 1968 to Q4 1977.

*Dummy variable used to represent the major automobile strike in Q4 1970.

TABLE 2

INDUSTRIES GROUPED ACCORDING TO THE VALUE OF b_3

High Values of b_3	Textile Mill Products (22)
.7 and higher	Paper and Allied Products (26)
High implied adjustment costs	Fabricated Metal Products (34)
Expectations important	Machinery, exc. electrical (35)

Lower, but Significant Values of b_3	Stone, Clay, and Glass (32)
	Primary Metals (33)
$b_3 = .3$ to $.5$	Electrical Equipment and (36) Supplies

Insignificant b_3	Apparel and Other
Low implied adjustment costs	Textile Products (23)
	Rubber and Plastic Products (30)
Expectations unimportant	Leather and Leather Products (31)
	Transportation Equipment (37)

The estimates of b_2 $\left(= \frac{dq}{\phi} \text{ in equation } (13) \right)$ are not signifi-
cant by a t-test at the 5 percent level. This, however, is
purely a consequence of collinearity with b_3, for F tests of
the hypothesis that output does not enter demand indicated
rejection for every industry.

The coefficient b_4 should always be greater than one and from
the theory should equal one plus the rate of interest.
Though b_4 is always positive, it is estimated with wide con-
fidence bounds and substantial variation across equations
which sometimes result in values below one.[1]

The summary statistics are of interest primarily for compari-
sons with the partial adjustment model, a special case in
which expectations are static. For a comparison of the two
models, the results of estimating a (linear) partial adjust-
ment equation are presented in table 3. The comparison
suggests the superiority of the expectations model: the R^2
and Durbin-Watson statistics are greater in every case for
the expectations model.

The familiar finding of strongly increasing returns to labor
(i.e., elasticities much less than one) is evident in the
elasticities calculated from the linear equation (partial
adjustment model). The elasticities, shown in table 4, are
all less than one and range from .483 (industry 37) to .792
(industry 36). The simple average over all industries is
.644. The nonlinear equation incorporating expectations
leads to elasticities that are higher in every industry than
those found in the partial adjustment model. In those

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[1]Appendix D provides somewhat more detail on the imprecision
of b_4, the estimated coefficient of 1+r.

TABLE 3

LABOR DEMAND REGRESSION EQUATIONS BY INDUSTRY
(Current Output Only)

$$N_{t+1} = a_1 + a_2 Q_{t+1} + a_3 N_t + a_4 D1 + a_5 D2 + a_6 D3 + a_7 TIME + a_8 D37*$$

Coef-ficient			Industry			
	22	23	26	30	31	32
a_1	120 (1.70)	134 (1.12)	161. (2.84)	222 (6.08)	12.2 (4.06)	116 (4.56)
a_2	3.38 (4.19)	7.18 (6.43)	2.00 (2.71)	6.84 (7.05)	3.56 (2.93)	6.84 (8.38)
a_3	.654 (7.24)	.603 (8.20)	.523 (3.53)	−.144 (−.109)	.802 (9.03)	.300 (3.66)
a_4	−2.83 (−.402)	−44.6 (−5.64)	−6.08 (−1.42)	−10.7 (−1.60)	−4.99 (−2.11)	21.3 (6.82)
a_5	4.64 (.666)	−25.7 (−3.09)	4.01 (.793)	−20.1 (−2.76)	1.92 (.831)	22.8 (5.51)
a_6	19.22 (2.64)	−35.2 (−4.59)	4.69 (1.18)	14.4 (2.28)	−2.72 (−1.21)	17.1 (5.87)
a_7	−.859 (−2.87)	.383 (1.63)	−1.24 (−2.85)	−1.55 (−4.23)	.214 (.750)	−.901 (−6.70)
R^2	.840	.934	.847	.893	.977	.950
DW	.936	1.42	.615	1.98	1.11	1.34

The range of the regressions is the same as in the previous labor demand regressions.

*Dummy variable used to represent the major automobile strike in Q4 1970.

TABLE 3 (Cont'd)

$$N_{t+1} = a_1 + a_2 Q_{t+1} + a_3 N_t + a_4 D1 + a_5 D2 + a_6 D3 + a_7 TIME + a_8 D37*$$

Coef-ficient	Industry				
	33	34	35	36	37
a_1	394. (4.77)	216. (3.95)	170. (2.50)	294. (3.68)	498. (3.63)
a_2	3.23 (5.68)	4.42 (7.89)	2.76 (4.92)	5.19 (6.63)	1.60 (2.74)
a_3	.313 (2.75)	.366 (4.40)	.620 (7.62)	.353 (3.59)	.455 (3.26)
a_4	-8.71 (-.860)	-13.6 (-1.82)	-23.3 (-1.83)	12.4 (.953)	-17.4 (-.918)
a_5	-17.0 (-1.52)	-24.5 (-2.67)	-38.1 (-3.06)	3.31 (2.52)	-31.9 (-1.53)
a_6	10.8 (1.07)	20.6 (2.87)	8.17 (.643)	67.5 (5.17)	59.3 (2.98)
a_7	-2.98 (-5.31)	.151 (.677)	-1.06 (-1.98)	-4.97 (-5.59)	-4.63 (-2.98)
a_8	-- --	-- --	-- --	-- --	-162.7 (-3.64)
R^2	.919	.934	.930	.915	.860
DW	1.03	1.49	.592	1.53	1.81

The range of the regressions is the same as in the previous labor demand regressions.

*Dummy variable used to represent major automobile strike in Q4 1970.

TABLE 4

LONG RUN ELASTICITIES OF LABOR
WITH RESPECT TO OUTPUT

Industry	e_1 (linear equation)	e_2 (nonlinear equation)
22	.659	1.008
23	.741	.835
26	.503	.684
30	.573	.660
31	.621	.654
32	.712	.732
33	.562	.733
34	.685	1.396
35	.757	.971
36	.792	.845
37	.483	.512
Average	.644	.821

$$e_1 = \frac{a_2}{1-a_3} \cdot \frac{\overline{Q}}{\overline{N}} \qquad e_2 = \frac{b_2 \cdot \Sigma_i b_3^i}{1-b_3 \cdot b_4} \cdot \frac{\overline{Q}}{\overline{N}}$$

where the a_i's are the coefficients from the linear labor demand equation

the b_i's are the coefficients from the nonlinear labor demand equation

and $\overline{Q}$, $\overline{N}$ are the means of output and employment, respectively.

industries where b_3 was significant, the average is .91 which
implies near constant returns. This result is obtained even
though the labor input is measured by the number of workers.
This contrasts with Sim's findings of increasing returns to
workers. In the other industries, taking account of the
future did not appreciably improve the estimated returns to
scale.

AUTOCORRELATION

That the nonlinear model improves the Durbin-Watson statistic
seems to indicate that introducing expectations accounts for
one source of autocorrelation. This accords with the inter-
pretation of autocorrelation as the consequence of omitted
variables (in this case, expectations) which are themselves
autocorrelated (see Madalla [9] p. 274).

Because both the linear and nonlinear equations contain a
lagged dependent variable the coefficients, t-values, and the
Durbin-Watson statistic are subject to bias. Whether it is
worthwhile trying to do anything about autocorrelation is
unclear. One view is that it is preferable not to perform a
correction but to use a measure of autocorrelation, such as
the Durbin-Watson or an estimated ρ to indicate the extent to
which there remain problems of omission or specification.
Further, Maeshiro [10] has shown that in small samples with
trended explanatory variables, GLS can frequently lead to a
greater mean square error and even greater bias, because of
increased multicollinearity.

It may still be instructive to consider the results of ad-
justing the nonlinear equation for autocorrelation. For 6 of
the industries, this adjustment makes little difference. In
3 of the 6 (industries 30, 31, and 37) the estimated b_3 was
still insignificantly different from 0 and so the simple
partial adjustment model would suffice. In the other 3
(industries 32, 34, and 36) the estimated ρ was small and
insignificant; the results presented in table 1 continue to
be appropriate.

Results for the other five industries, where the adjustment
does make a difference, are reported in table 5. For 2
industries, 22 and 33, the sharpest changes have to do with
decreases in the value of b_4 (the estimate of $1+r$). This
finding is not unexpected when correcting for autocorrelation
since b_4 is the coefficient on the lagged dependent variable.
The estimated covariances imply that it is also, however, a
consequence of added multicollinearity problems arising from
the extra coefficient (ρ) to be estimated (since ρ and b_4 are
somewhat collinear). In a third industry, 23, the coeffi-
cients b_3 and b_4 increase in value and both now become signi-
ficant. In all 3 industries, in spite of slight changes in
the values of b_3 and b_4 the elasticities are similar to
those calcualted earlier when ρ was not estimated.

In two industries, 26 and 35, estimating ρ induced a lack of
convergence in the nonlinear routine, possibly because of
added multicollinearity. For industry 26, the lack of con-
vergence was overcome by including a dummy variable for the
final quarter of 1974 and the first quarter of 1975. These
were quarters of extreme labor dishoarding to an extent in-
consistent with our model; they would make a good base for
future work on the conditions when hoarding breaks down.

For industry 35, the problem of nonconvergence could be
solved by including relative wage terms.[1] This required
modifying the cost function (5). Otherwise the first order
conditions lead to a second-order difference equation (given
in equation (7)) with nonconstant coefficients due to the
term $\frac{cq_t}{\phi}$.

---

[1] In general, however, when relative wages were included in
the industry labor demand equations they did not improve the
regression results.

TABLE 5

LABOR DEMAND REGRESSION EQUATIONS BY INDUSTRY
(Current and Expected Output)
(Correction for Autocorrelation)

$$N_{t+1} = b_1 + b_2 \sum_{i=1}^{9} b_3^i Q_{t+i}^* + b_4 b_3 N_t + b_5 \text{ TIME} + b_6 D1 + b_7 D2$$

$$+ b_8 D3 + b_9 D37 + b_{10} D26 + b_{11} \sum_{i=1}^{9} b_3^i W_{t+i}^*$$

Coef-ficient	Industry		
	22	23	26
b_1	60.8 (.367)	96.2 (.66)	133.2 (1.82)
b_2	2.67 (.827)	7.26 (1.01)	1.82 (1.03)
b_3	.751 (2.62)	.516 (2.28)	.505 (2.39)
b_4	.638 (2.74)	1.18 (2.29)	1.15 2.83)
b_5	-1.88 (-1.85)	.569 (.957)	-1.04 (-3.17)
b_6	-3.62 (.995)	-40.86 (-6.24)	-4.39 (-2.07)
b_7	6.29 (1.42)	-14.26 (-1.58)	.513 (1.87)
b_8	13.1 (2.17)	-26.84 (-4.3)	2.17 (1.18)
b_9	--	--	--
b_{10}	-- --	-- --	-28.8 (-7.44)
b_{11}	--	--	--
ρ	.769 (4.54)	.433 (1.61)	.18 (.76)

TABLE 5 (Cont'd)

$$N_{t+1} = b_1 + b_2 \sum_{i=1}^{9} b_3^i Q_{t+i}^* + b_4 b_3 N_t + b_5 \text{ TIME} + b_6 D1 + b_7 D2$$

$$+ b_8 D3 + b_9 D37 + b_{10} D26 + b_{11} \sum_{i=1}^{9} b_3^i W_{t+i}^*$$

Coef- ficient	33	Industry 35
b_1	448. (2.90)	884. (2.27)
b_2	9.05 (1.85)	4.25 (1.60)
b_3	3.51 (2.29)	.535 (3.93)
b_4	2.81 (.736)	.576 (1.45)
b_5	-4.24 (-4.58)	-3.44 (-2.13)
b_6	-12.84 (-1.84)	-10.9 (-1.54)
b_7	-7.47 (-.728)	-18.1 (-2.46)
b_8	17.68 (2.54)	6.31 (.704)
b_9	-- --	-- --
b_{10}	-- --	-- --
b_{11}	-- --	-439.5 (-1.58)
ρ	.605 (3.86)	.677 (2.91)

We adopt a variant of the cost function discussed by Sims and Kennan which assumes that costs are made up of disequilibrium (away from equilibrium level labor, N*) and adjustment costs, and is given by

$$C_{t+j} = \frac{\alpha}{2} \left(N_{t+j} - N^*_{t+j}\right)^2 + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j}\right)^2 . \tag{19}$$

Using this in the expected discounted cost function leads to a labor demand equation quite similar to equation (13) except that λ_1 now equals $\dfrac{2h'a}{1 + \sqrt{1 - 4(h')^2 a}}$ where $\left(h'\right)^{-1} = \dfrac{\alpha + (2+r)\phi}{\phi}$

(previously, $h^{-1} = \dfrac{cq + (2+r)\phi}{\phi}$ so that α substitutes for cq in the definition of h).

The results for industries 26 and 35 are included in table 5. The coefficient b_3 was quite significant in both cases although the magnitude fell somewhat from the previous regressions.

In general, the autocorrelation correction did not change the qualitative results; the same set of seven industries shows a significant dependence between labor demand and rational expectations of the future and one industry (23) is added to the set.

A COMPARISON OF SHORT-RUN AND LONG-RUN ELASTICITIES

This section describes how the theoretical labor demand model and empirical output model may be used to determine how changes in imports and GNP affect labor demand. We wish to determine the extent to which changes in current imports lead to changes in expected imports, which then lead to changes in expected output and then, finally, to changes in employment.

Although the results of this calculation are of interest for public policy reasons, it also illustrates the usefulness of the labor demand model that incorporates expectations of the future. For example, in the present case, it is used to distinguish between different sources of output change. Current output will change by the same amount regardless of whether the change comes about due to changes in imports or GNP. The effect on what is _expected_ to happen will be quite different, however.

An increase in imports in a particular industry may be viewed by domestic producers as requiring a permanent decrease in that industry's domestic production. If so, it should cause nearly immediate adjustment in employment. Alternatively, imports may be small enough relative to industry demand, or contain enough random variation, that firms do not adjust quickly.

The Calculation of Short-Run Elasticities

In order to estimate the speed of adjustment of employment to imports and compare it to other sources of output change, the model is used to evaluate the derivative of employment with respect to current output. Whereas, the long-run elasticity assumes a steady state for N and Q and is calculated as

$$e_L = \frac{dN}{dQ} \frac{\overline{Q}}{\overline{N}} ,$$

the short-run elasticity uses current values N_0 and Q_0 and is calculated as

$$e_S = \frac{dN_0}{dQ_0} \frac{\overline{Q}}{\overline{N}} .$$

The quantity of interest is the derivative dN_0/dQ_0. In the model, this derivative will incorporate the effect of current output on expected output, which then affects current employment. Alternative calculations are made for a change in current output attributable to imports and a change attributable to a change in GNP. In both cases, the decrease in current output is identical. Calculated differences in the response of employment were due to what changes in output firms expected in the future. If changes in current imports represent a permanent change, the effect on expected output will be greater. In turn, employment should respond faster to a given change. Calculations are only made for those industries where expectations of output are important (a significant b_3).

The derivative of current employment with respect to current output (via a change in imports) was estimated by totally differentiating the nonlinear model. The change in labor demand arises from a change in imports, both current and expected, leading to changes in domestic output.

The equations were of the following form:

$$dlnM_i = g_1 dlnM_{i-1} + g_2 dlnM_{i-2} \ , \ i=1,\ldots,8$$

$$dQ_i = -dM_i \ , \ i=1,\ldots,8$$

$$dN_o = b_2 \sum b_3^i dQ_i$$

The current period is signified by the subscript o. The subscript i denotes the number of periods in the future. The equations themselves are the total differential of the estimated equations in the model. Thus, the coefficients g_1 and g_2 are the coefficients in the import equation where two lags in imports are the important explanatory variables. A change in current imports leads to the forecasts incorporating this change through these coefficients. This equation denotes a number of equations referring to expectations at different points in the future.

$$dlnM_1 = g_1 dlnM_o$$

$$dlnM_2 = g_1 dlnM_1 + g_2 dlnM_o$$

$$dlnM_3 = g_1 dlnM_2 + g_2 dlnM_1$$

.
.
.

With an initial value for dM_0 equal in amount to the value obtained from a 5 percent change in GNP, these and the previous equations are used successively to solve $dM_1, \ldots, dM_8$, then $dQ_0, \ldots, dQ_8$, and finally dN_0. From this, dN_0/dQ_0 is calculated and then converted to an elasticity at the sample means.

In order to estimate the corresponding derivative that
results from a change in GNP, we use a similar procedure.
The equations were of the form:

$$dlnGNP_i = f_1 dlnGNP_{i-1} + f_2 dlnGNP_{i-2}, \quad i=1,\ldots,8$$

$$dlnQ_i = h_1 dlnGNP_i + h_2 dln \left(\frac{GNP_i}{GNP_{i-1}} \right), \quad i=1,\ldots,8$$

$$dN_o = b_2 \sum b_3^i dQ_i .$$

Once the alternative derivatives of employment with respect
to current output were evaluated, they were then converted to
an elasticity. The elasticities for the seven industries are
reported in table 6. The symbols e_G and e_M denote the short-
run elasticities derived when GNP and imports change, respec-
tively. The symbol e_2 is the long-run elasticity derived
earlier and repeated here for convenience. Also, an adjust-
ment parameter (or speed of adjustment), η, is obtained by
dividing the short-run elasticity by the long-run elasticity
in each case (also denoted by G or M). The adjustment
parameter is the fraction of adjustment completed by the firm
in each period, toward the equilibrium level N^*. The param-
eter is analogous to the parameter η in the simple partial
adjustment model

$$N_t - N_{t-1} = \eta \left[N_t^* - N_{t-1} \right]$$

As expected, short-run elasticities are all lower than the
long-run elasticities. More surprisingly, in every industry,
the short-run change in labor demand is greater for a change
in GNP than for imports. Apparently, even though changes in
GNP may be thought of by the firm as cyclical changes, and

TABLE 6

SHORT-RUN ELASTICITIES AND ADJUSTMENT PARAMETERS
FOR A CHANGE IN GNP and IMPORTS

Industry	e_G	e_M	e_L	Adjustment Parameter η_G	η_M
22	.256	.113	1.008	.254	.113
26	.314	.140	.684	.459	.205
32	.636	.380	.732	.869	.519
33	.483	.326	.733	.659	.431
34	.726	.273	1.396	.520	.196
35	.474	.089	.971	.488	.092
36	.638	.386	.845	.755	.457

therefore only "temporary," the decrease in labor demand in response to the change is greater and occurs more rapidly. The adjustment of labor to GNP occurs substantially more quickly than the adjustment to a higher level of imports.

CONCLUSIONS

Our most important results are summarized below.

(1) Expectations of the future, though typically omitted
from empirical studies of labor demand, have an important
effect and should be incorporated explicitly. This is true
even for the fairly simple output model developed earlier.
In particular, we found that for eight of the eleven two-
digit industries studied, expectations, as measured assuming
rationality, had a significant effect on current labor
demand.

(2) The incorporation of expectations tends to reduce
measured economies of scale, so that the estimated long-run
labor-output elasticity is closer to one. This raises the
possibility that any remaining deviation from constant
returns is also a consequence of omitted variable bias.

(3) It does seem feasible to distinguish empirically the
effects of different sources of output change. In our case,
we started with the hypothesis that changes in imports would
induce more rapid adjustment than changes in GNP, but the
empirical evidence pointed in the opposite direction.

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APPENDIX A

DERIVATION OF THE DEMAND FOR LABOR

APPENDIX A

DERIVATION OF THE DEMAND FOR LABOR

Appendix A presents a more complete description of the
derivation in the text using the methodology proposed in
Sargent [17]. This methodology is of interest primarily for
two reasons: (1) much of our work is based on Sargent's past
work on rational expectations and (2) in his recent work he
proposed a model quite similar to the one already discussed.
In that model, he uses a quadratic adjustment cost function
coupled with a quadratic single-input production function.

This appendix consists of two sections. The first is a
general discussion of lag operators which are used in the
derivation. A lag operator is one which when taken to the
n^{th} power and multiplied by a variable, X, gives the value of
X shifted back n periods. The second section is the
alternative derivation of the labor demand equation to be
used in the empirical work.

LAG OPERATORS

We shall consider polynomials in the lag operator

$$A(L) = a_0 + a_1 L + a_2 L^2 + \ldots = \sum_{j=0}^{\infty} a_j L^j \qquad (A-1)$$

where the a's are constants and $LX_t = X_{t-1}$ and $L^n X_t = X_{t-n}$. Then

$$A(L)X_t = a_0 X_t + a_1 X_{t-1} + \ldots = \sum_{j=0}^{\infty} a_j X_{t-j}.$$ The polynomials

A(L) that will be considered in this paper are "rational" in
the sense that they can be expressed as the ratio

$$A(L) = B(L)/C(L) \tag{A-2}$$

where

$$B(L) = \sum_{j=0}^{m} b_j L^j, \quad C(L) = \sum_{j=0}^{n} c_j L^j$$

and the b_j's and c_j's are constants.

For a simple example of a rational polynomial in L and one which will become important later in this section, consider

$$A(L) = \frac{1}{1-\lambda L} . \tag{A-3}$$

As long as $|\lambda| < 1$, there is the following useful expansion

$$\frac{1}{1-\lambda L} = 1 + \lambda L + \lambda^2 L^2 + \ldots \tag{A-4}$$

If $|\lambda| > 1$, or equivalently where $\left|\frac{1}{\lambda}\right| < 1$, an alternative expansion become useful for the polynomial $1/(1-\lambda L)$. That is,

$$\frac{1}{1-\lambda L} = \frac{-(\lambda L)^{-1}}{1-(\lambda L)^{-1}} = \frac{-1}{\lambda L}\left(1 + \frac{1}{\lambda} L^{-1} + \left(\frac{1}{\lambda}\right)^2 L^{-2} + \ldots\right)$$

$$= -\frac{1}{\lambda} L^{-1} - \left(\frac{1}{\lambda}\right)^2 L^{-2} - \left(\frac{1}{\lambda}\right)^3 L^{-3} - \ldots \tag{A-5}$$

Equation (A-5) implies that

$$\frac{1}{1-\lambda L} X_t = -\frac{1}{\lambda} X_{t+1} - \left(\frac{1}{\lambda}\right)^2 X_{t+2} - \ldots = -\sum_{i=1}^{\infty} \left(\frac{1}{\lambda}\right)^i X_{t+i}$$

which says that $(1/(1-\lambda L))X_t$ is a geometrically declining weighted sum of future values of X.

THE DEMAND FOR LABOR

The sum of discounted costs is given by the following equation:

$$V_t = \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \frac{1}{a_{t+i}} \right) C_{t+j} \quad \text{where} \quad a_{t+i} = 1 + r_{t+i}$$

and

$$
\begin{aligned}
C_{t+j} &= W_{t+j} N_{t+j} q_{t+j} Z_{t+j} + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j} \right)^2 \\
&= W_{t+j} N_{t+j} + q_{t+j} \left(a + b N_{t+j} + \frac{c}{2} N_{t+j}^2 \right. \\
&\quad \left. + d N_{t+j} Q_{t+j} + e Q_{t+j} + f Q_{t+j}^2 \right) + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j} \right)^2
\end{aligned}
$$

The firm is assumed to know the sequences

$$\left\{ W_{t+j} \right\}_{j=0}^{\infty} \quad \text{and} \quad \left\{ Q_{t+j} \right\}_{j=0}^{\infty} \quad \text{and chooses a sequence}$$

$$\left\{ N_{t+j} \right\}_{j=0}^{\infty} \quad \text{to maximize } V_t[1]. \text{ For simplicity,}$$

an additional assumption is made that the sequences

$$\left\{ q_{t+j} \right\}_{j=0}^{\infty} \quad \text{and} \quad \left\{ a_{t+j} \right\}_{j=0}^{\infty} \quad \text{are constant over time}$$

and equal q and a, respectively. Further, it is assumed for all t that for some K>0, $|N_t| < Kx^t$, where $1<x<a$; sequences that satisfy these inequalities for some K>0 and $1 \le x < a$ will be termed of exponential order less than a (this terminology is due to Sargent).

The first order conditions to the cost minimization problem are given by

---

[1] In this and the next appendix, we have dropped the expectation operator, E_t, and assume that $\left\{ W_{t+j} \right\}$ and $\left\{ Q_{t+j} \right\}$ are known with certainty to simplify the derivation. To show that the solution to the quadratic discounted cost function exhibits "certainty equivalence," see [8] or [17].

$$\frac{\partial V_t}{\partial N_{t+j}} = \phi\left(N_{t+j} - N_{t+j-1}\right) \frac{1}{a_t \cdots a_{t+j-1}} \qquad (A-7a)$$

$$+ \left[W_{t+j} + q_{t+j}\left(b+cN_{t+j} + dQ_{t+j}\right)\right.$$

$$\left. - \phi\left(N_{t+j+1} - N_{t+j}\right)\right] \frac{1}{a_t \cdots a_{t+j}}$$

$$= 0, \text{ for } j=0,1\ldots,T-1, \text{ and}$$

$$\frac{\partial V_t}{\partial N_{t+T}} = \frac{1}{a_t \cdots a_{t+T}} \; \phi\left(N_{t+T+1} - N_{t+T}\right) = 0 \text{ for } j=T. \quad (A-7b)$$

After some rearrangement, equation (A-7a) is equal to

$$N_{t+j} = h_{t+j}\left(a_{t+j}N_{t+j-1} - \frac{X_{t+j}}{\phi} + N_{t+j+1}\right)$$

where $h_{t+j} = h = \dfrac{1}{2+r+\dfrac{cq}{\phi}}$ and $X_{t+j} = W_{t+j}+bq+dqQ_{t+j}$

It will turn out to be convenient to rearrange this equation as

$$N_{t+j+1} - \frac{1}{h} N_{t+j} + aN_{t+j-1} = \frac{1}{\phi} X_{t+j} \quad j=0,1,\ldots T-1 \quad (A-8)$$

Two boundary conditions are needed to solve this second order difference equation. The first is given by the initial level of labor, i.e., N_{t-1}. The second is given by the terminal condition (A-7b). That condition is known as the transversality condition and is necessary for optimality. Taking the limit as $T\to\infty$, it is given by

$$\lim_{T\to\infty} \left(\frac{1}{a}\right)^{T+1} \phi\left(N_{t+T+1} - N_{t+T}\right) = 0. \qquad (A-9)$$

A sufficient condition for this to hold is that the solution sequence of the N_{t+j}'s be of exponential order less than a. To show sufficiency, note that

$$\left(\frac{1}{a}\right)^{T+1} \phi\left(N_{t+T+1}-N_{t+T}\right) \leq \left(\frac{1}{a}\right)^{T+1} |N_{t+T+1}| + \left(\frac{1}{a}\right)^{T+1} |N_{t+T}|$$

$$\leq \left(\frac{1}{a}\right)^{T+1} Kx^{t+T+1} + \left(\frac{1}{a}\right)^{T+1} Kx^{t+T}$$

$$= Kx^t \left(\frac{x}{a}\right)^{T+1} + Kx^{t-1}\left(\frac{x}{a}\right)^{T+1}$$

Previously, it was given that $\frac{x}{a} < 1$ (since $x < a$). This means that the limit of both terms as $T \to \infty$ is zero and the transversality condition (A-9) is satisfied.

The necessary conditions for optimality for the infinite horizon problem are satisifed if a solution to difference equation (A-8) is found subject to the initial value N_{t-1} and the transversality condition (A-9).

To arrive at a solution for (A-8), we begin by writing it as

$$\left(1 - \frac{1}{h} L + aL^2\right)N_{t+j+1} = \frac{1}{\phi} X_{t+j} \tag{A-10}$$

It is useful to write the polynomial $1 - \frac{1}{h} L + aL^2$ in an alternative way, given by what is termed the "factorization"

$$1 - t_1 L - t_2 L^2 = \left(1 - \lambda_1 L\right)\left(1 - \lambda_2 L\right) \tag{A-11}$$

$$= 1 - \left(\lambda_1 + \lambda_2\right)L + \lambda_1 \lambda_2 L^2$$

where $t_1 = \lambda_1 + \lambda_2 = \frac{1}{h}$ and $t_2 = -\lambda_1 \lambda_2 = -a$.

The l.h.s. of (A-11) is the polynomial $1 - t_1 z - t_2 z^2$.

Notice that

$$\left(1 - \lambda_1 z\right)\left(1 - \lambda_2 z\right) = \lambda_1 \lambda_2 \left(\frac{1}{\lambda_1} - z\right)\left(\frac{1}{\lambda_2} - z\right). \qquad (A-12)$$

When set equal to 0, this equation is satisfied at the two roots $z = 1/\lambda_1$ and $z = 1/\lambda_2$. The characteristic equation $1 - t_1 z - t_2 z^2 = 0$, could be solved for two values of z.

Before calculating the solution to the second order difference equation, however, let us follow Sargent and provide a more heuristic solution. Remember that

$$\lambda_1 + \lambda_2 = \frac{1}{h} = a + \frac{\phi + cq}{\phi} \quad \text{where} \quad \frac{\phi + cq}{\phi} > 1. \qquad (A-13)$$

and

$$\lambda_1 \lambda_2 = a \quad \text{or} \quad \lambda_2 = a/\lambda_1. \qquad (A-14)$$

Equation (A-13) means that λ_1 must satisfy

$$\frac{1}{h} = \lambda_1 + a/\lambda_1$$

$$\text{or} \quad a + \frac{\phi + cq}{\phi} = \lambda_1 + a/\lambda_1 = f\left(\lambda_1\right)$$

Figure 1 is used to help illustrate how λ_1 and λ_2 are determined. First, the function $f(\lambda)$ achieves a minimum at $\lambda = \sqrt{a}$ and is equal to $2\sqrt{a}$ at this point. For any $a > 1$, or equivalently, for any $r > 0$, it can be shown that $1 + a > 2\sqrt{a}$. The function $f(\lambda)$ equals $1 + a$ when $\lambda = 1$ and $\lambda = a$. This means that the solutions for λ_1 and λ_2 are real and distinct and as implied in the picture. We have assumed (w.l.o.g.) that $\lambda_1 < \lambda_2$, or more specifically, that $\lambda_1 < \sqrt{a} < \lambda_2$. Since $\frac{1}{h} = a + \frac{\phi + cq}{\phi}$, then as long as $\frac{cq}{\phi} > 0$, $\frac{1}{h} > 1 + a$. This implies that $\lambda_1 < 1$ and $\lambda_2 > a$.
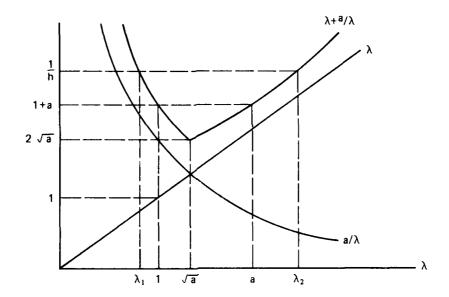
FIG. 1: GRAPHIC REPRESENTATION OF THE DETERMINATION OF λ

We are now able to use the factorization and write equation (A-8) as

$$\left(1-\lambda_1 L\right)\left(1-\lambda_2 L\right)N_{t+j+1} = \frac{1}{\phi} X_{t+j} \ .$$

In order to satisfy the transversality condition it is necessary to use the forward inverse of $\left(1-\lambda_2 L\right)$ (see equation (A-5) in the lag operators section) or

$$\left(1-\lambda_1 L\right)N_{t+j+1} = -\frac{\left(\phi \lambda_2\right)^{-1}L^{-1}}{1-\lambda_2^{-1}L^{-1}} \ X_{t+j} \ . \qquad (A-15)$$

From the relationship $\lambda_2 = a/\lambda_1$, we may rewrite this expression as

$$\left(1-\lambda_1 L\right)N_{t+j+1} = -\frac{\lambda_1}{a\phi} \ \frac{1}{1-\lambda_2^{-1}L^{-1}} \ X_{t+j+1} \qquad (A-16)$$

$$\left(\text{since } L^{-1}X_{t+j} = X_{t+j+1}\right) \ .$$

A-7

Writing the denominator as

$$\left(1 - \lambda_2^{-1} L^{-1}\right)^{-1} = 1 + \lambda_2^{-1} L^{-1} + \lambda_2^{-2} L^{-2} + \ldots$$

and using this expression in equation (A-16) leads to the equation for labor demand in the t+j+1 st period:

$$N_{t+j+1} = \lambda_1 N_{t+j} - \frac{\lambda_1}{a\phi} \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i X_{t+j+1+i}$$

or   (A-17)

$$N_{t+j+1} = \lambda_1 N_{t+j} - \frac{\lambda_1}{a\phi} \sum_{i=0}^{\infty} \left(\frac{\lambda_1}{a}\right)^i X_{t+j+1+i}$$

Although we have derived a range on λ_1 and λ_2, an explicit value in terms of the production function parameters, interest rate, other factor input costs, and the adjustment cost parameter has not been given. In order to do this now, it is necessary to use the characteristic equation given by $1 - t_1 z - t_2 z^2 = 0$, where it was previously stated that $t_1 = 1/h$ and $t_2 = -a$. The roots are given by the quadratic formula

$$z = \frac{-t_1 \pm \sqrt{t_1^2 + 4t_2}}{2t_2}$$

Substituting in the values for t_1 and t_2 leads to

$$z = \frac{-\frac{1}{h} \pm \sqrt{\frac{1}{h^2} - 4a}}{-2a} = \frac{-\frac{1}{h} \pm \sqrt{\frac{1 - 4h^2 a}{h^2}}}{-2a}$$

$$= \frac{-\frac{1}{h} \ 1 \pm \sqrt{1 - 4h^2 a}}{-2a}$$

or finally,

$$z = \frac{1 \pm \sqrt{1-4h^2a}}{2ha} \; .$$

Remember that z is equal to the reciprocals of λ_1 and λ_2.

Therefore, when

$$z = \frac{1 + \sqrt{1-4h^2a}}{2ha}$$

then

$$\lambda_1 = \frac{1}{z} = \frac{2ha}{1+ \sqrt{1-4h^2a}} \; ,$$

and an expression for equation (A-17) is obtained.[1]

---

[1] The same can be shown for $z = \frac{1}{\lambda_2}$ where $\lambda_2 = \frac{1- \sqrt{1-4h^2a}}{2ha}$. It can be shown that $\frac{1}{\lambda_2} < 1$ and is equivalent to $\frac{\lambda_1}{a}$ (substitute in for the values of λ_1 and λ_2 and use the relationship $\lambda_1 \lambda_2 = a$ and it is derived quite simply).

APPENDIX B

A SECOND DERIVATION OF THE DEMAND FOR LABOR

APPENDIX B

A SECOND DERIVATION OF THE DEMAND FOR LABOR

The model includes a simple quadratic adjustment cost function in a framework similar to the one developed by Brechling [1]. It will turn out that the firm's demand for labor in the present period depends upon the entire future path of the exogeneous variables. In particular, the exogeneous variable of most interest is the output measure.

THE ASSUMPTIONS

Turning now to the development of the model, a standard production function is assumed where output in period t is produced by two factor inputs--labor and aggregate input Z (which has already been briefly discussed):

$$Q_t = F\left(N_t, Z_t\right), \quad F_N > 0, \quad F_Z > 0, F_{NN} < 0, \quad F_{ZZ} < 0. \quad (3-1)$$

N may be thought of as the number of men at work or the total number of man-hours; we assume it refers to the number of men. The firm is assumed to be a cost minimizer and takes output as given. Rather than use the production function, we assume that the inverse production function (in terms of Z_t) exists. This presents no problem, for example, if a Cobb-Douglas or CES production function is assumed. These functions are not suitable for optimization over time, so we work with a quadratic expression around the steady state (Q*, N*) of the inverse production function $G\left(Q_t, N_t\right)$. This quadratic approximation can be interpreted as Taylor's expansion:

$$Z_t = G\left(Q_t^* + \Delta_1, \ N_t^* + \Delta_2\right) = G\left(Q_t^*, \ N_t^*\right) \tag{B-2}$$

$$+ \left(Q_t \ \frac{\partial G}{\partial \Delta_1} + N_t \ \frac{\partial G}{\partial \Delta_2}\right) + \frac{1}{2}\left(Q_t^2 \ \frac{\partial^2 G}{\partial \Delta_1^2} + N_t^2 \ \frac{\partial^2 G}{\partial \Delta_2^2}\right.$$

$$\left. + \ 2N_t Q_t \ \frac{\partial^2 G}{\partial \Delta_1 \partial \Delta_2}\right) + \ \ldots$$

$$= a + bN_t + \frac{c}{2} N_t^2 + dN_t Q_t + eQ_t + fQ_t^2 + \ldots$$

The signs of the coefficients c and d will turn out to be important later on; it is necessary that

$$c > 0, \ d < 0. \tag{B-3}$$

For Cobb-Douglas or CES production functions,[1] the second order conditions assure that these are met.

The firm is assumed at time t to minimize the sum of discounted future costs or payments to factors (including adjustment costs) where this cost is represented for the t+jth period as,

$$C_{t+j} = W_{t+j}N_{t+j} + q_{t+j}Z_{t+j} + \frac{\phi}{2}\left(N_{t+j+1} - N_{t+j}\right)^2 \tag{B-4}$$

$$= W_{t+j}N_{t+j} + q_{t+j}\left(a + bN_{t+j} + \frac{c}{2}N_{t+j}^2 + dN_{t+j}Q_{t+j} + eQ_{t+j}\right.$$

$$\left. + \ fQ_{t+j}^2\right) + \frac{\phi}{2}\left(N_{t+j+1} - N_{t+j}\right)^2$$

This flow of costs is discounted by interest rates $\left(r_t, \ldots, r_{t+T}\right)$ and summed over periods $j = 0, \ldots, T$. To find the optimal plan for the firm note that the only choice variable is N_{t+j}, since Q_{t+j} is given to the firm and Z_{t+j} is assumed to adjust instantaneously to produce this output according to the production function.

_____

[1] Although this can be made more general.

Focusing on three terms in the sum of discounted costs, namely those for periods t+j-1, t+j, and T gives:

$$V_t' = \ldots + \left[W_{t+j-1} N_{t+j-1} + q_{t+j-1} \left(a + bN_{t+j-1} + \frac{c}{2} N_{t+j-1}^2 \right.\right.$$
$$\left.+ dN_{t+j-1} Q_{t+j-1} + eQ_{t+j-1} + fQ_{t+j-1}^2 \right)$$
$$+ \frac{\phi}{2} \left(N_{t+j} - N_{t+j-1} \right)^2 \left] \frac{1}{\left(1+r_t\right)\ldots\left(1+r_{t+j-1}\right)} \right. \quad (B-5)$$
$$+ \left[W_{t+j} N_{t+j} + q_{t+j} \left(a + bN_{t+j} + \frac{c}{2} N_{t+j}^2 + dN_{t+j} Q_{t+j} \right.\right.$$
$$\left.+ eQ_{t+j} + fQ_{t+j}^2 \right) + \frac{\phi}{2} \left(N_{t+j+1} - N_{t+j} \right)^2 \left] \frac{1}{\left(1+r_t\right)\ldots\left(1+r_{t+j}\right)} \right.$$
$$+ \ldots + \left[W_T N_T + q_T \left(a + bN_T + \frac{c}{2} N_T^2 + dN_T Q_T + eQ_T + fQ_T^2 \right) \right.$$
$$+ \frac{\phi}{2} \left(N_{t+T+1} - N_{t+T} \right)^2 \left] \frac{1}{\left(1+r_t\right)\ldots\left(1+r_{t+T}\right)} \right.$$

THE FIRST-ORDER CONDITIONS AND THE SOLUTION FOR LABOR DEMAND

Differentiating (B-5) with respect to N_{t+j} and N_{t+T} gives the following first order conditions:

$$\frac{\partial V_t'}{\partial N_{t+j}} = \phi \left(N_{t+j} - N_{t+j-1} \right) \frac{1}{\left(1+r_t\right)\ldots\left(1+r_{t+j-1}\right)} \quad (B-6)$$
$$+ \left[W_{t+j} + q_{t+j} \left(b + cN_{t+j} + dQ_{t+j} \right) \right]$$
$$- \phi \left(N_{t+j-1} + N_{t+j} \right) \frac{1}{\left(1+r_t\right)\ldots\left(1+r_{t+j}\right)} = 0$$

$$\frac{\partial V_t'}{\partial N_{t+T}} = \phi \left(N_{t+T+1} - N_{t+T} \right) \frac{1}{\left(1+r_t\right)\ldots\left(1+r_{t+T}\right)} = 0 \quad (B-6a)$$

Equation (B-6) must hold for all j between 0 and T. The next equation (B-6a) represents an end-point condition (transversality condition) and has been discussed in more detail in appendix A. Multiply equation (B-6) by $\left(1+r_t\right)\ldots\left(1+r_{t+j}\right)$ to obtain,

$$\phi\left(N_{t+j}-N_{t+j-1}\right)\left(1+r_{t+j}\right)+W_{t+j}+bq_{t+j}+cq_{t+j}N_{t+j}+dq_{t+j}Q_{t+j}$$
$$-\phi\left(N_{t+j+1}-N_{t+j}\right)=0$$

and collect terms for N_{t+j}

$$N_{t+j}\left[\left(1+r_{t+j}\right)\phi+cq_{t+j}+\phi\right]=\phi N_{t+j-1}\left(1+r_{t+j}\right) \qquad (B-7)$$
$$-W_{t+j}-bq_{t+j}-dq_{t+j}Q_{t+j}+\phi N_{t+j+1}.$$

Dividing by the adjustment cost ϕ, yields

$$N_{t+j}\left[\left(1+r_{t+j}\right)+\frac{cq_{t+j}}{\phi}+1\right]=N_{t+j-1}\left(1+r_{t+j}\right)$$
$$-\frac{W_{t+j}}{\phi}-\frac{bq_{t+j}}{\phi}-\frac{dq_{t+j}Q_{t+j}}{\phi}+N_{t+j+1}$$

or

$$N_{t+j}=\frac{1}{\left(1+r_{t+j}\right)+\frac{cq_{t+j}}{\phi}+1}\left[N_{t+j-1}\left(1+r_{t+j}\right)\right. \qquad (B-8)$$
$$\left.-\frac{W_{t+j}}{\phi}-\frac{bq_{t+j}}{\phi}-\frac{dq_{t+j}Q_{t+j}}{\phi}+N_{t+j+1}\right]^1$$

---

[1] Note that for $\phi=0$ (i.e., no adjustment costs) and after some rearrangement (B-8) implies the following:

$$N_{t+j}=\frac{-W_{t+j}-bq_{t+j}-dq_{t+j}Q_{t+j}}{cq_{t+j}}=\frac{-b}{c}-\frac{c'W_{t+j}}{q_{t+j}}-d'Q_{t+j}$$

where $c'=\frac{1}{c}>0$

$\qquad d'=\frac{d}{c}<0.$

In order to simplify the exposition, the following relationships are used throughout the paper. Let

$$h_{t+j} = h\left(r_{t+j}, \, q_{t+j}\right) = \frac{1}{2+r_{t+j}+\dfrac{cq_{t+j}}{\phi}} \qquad (B\text{-}9)$$

and

$$a_{t+j} = \left(1+r_{t+j}\right) \qquad (B\text{-}10)$$

Using (B-9) and (B-10), (B-8) now becomes

$$N_{t+j} = h_{t+j}\left[N_{t+j-1}\; a_{t+j} - \left(\frac{W_{t+j}+bq_{t+j}+dq_{t+j}Q_{t+j}}{\phi}\right) + N_{t+j+1}\right] \qquad (B\text{-}8a)$$

Furthermore, let $X_{t+j} = W_{t+j}+bq_{t+j}+dq_{t+j}Q_{t+j}$ leading to

$$N_{t+j} = h_{t+j}\left[N_{t+j-1}a_{t+j} - \frac{X_{t+j}}{\phi} + N_{t+j+1}\right] \qquad (B\text{-}8b)$$

which is a nonhomogeneous difference equation with non-constant coefficients (because X_t, N_t, and a_t will vary over time).

To derive a general solution to this equation, let us first assume that the present period is denoted by 1, last period by 0, and the future by 2,3,..., etc. At the end of the derivation, the more general notation will be used. The use of the simple notation is used only to illustrate the solution.

We begin by rewriting equation (B-8b) as

$$N_1 = h_1 a_1 N_0 - h_1 \; \frac{X_1}{\phi} + h_1 N_2 \qquad (B\text{-}11)$$

where N_0, last period's labor services, is known to the firm.

Similarly, for period 2,

$$N_2 = h_2 a_2 N_1 - h_2 \frac{X_2}{\phi} + h_2 N_3 \qquad (B-12)$$

From (B-11), we substitute for N_1, yielding

$$N_2 = h_2 a_2 \left[h_1 a_1 N_0 - h_1 \frac{X_1}{\phi} + h_1 N_2 \right] - h_2 \frac{X_2}{\phi} + h_2 N_3 \qquad (B-12a)$$

Solving for N_2, we have

$$N_2 = \frac{h_2 h_1 a_2 a_1}{1 - h_2 h_1 a_2} \; N_0 - \frac{h_2 h_1 a_2}{1 - h_2 h_1 a_2} \; \frac{X_1}{\phi} - \frac{h_2}{1 - h_2 h_1 a_2} \; \frac{X_2}{\phi}$$

$$+ \frac{h_2}{1 - h_2 h_1 a_2} \; N_3 \; . \qquad (B-12b)$$

For ease of exposition, define

$$y_t = \frac{h_t}{1 - h_t a_t y_{t-1}} \qquad (B-13)$$

where $y_0 = 0$.

Using the relationship in (B-12b) yields

$$N_2 = y_2 y_1 a_2 a_1 N_0 - y_2 y_1 a_2 \; \frac{X_1}{\phi} - y_2 \; \frac{X_2}{\phi} + y_2 N_3 \qquad (B-12c)$$

Substituting (B-12c) into (B-11), we have

$$N_1 = y_1 a_1 N_0 - y_1 \frac{X_1}{\phi} + y_1 \left[y_2 y_1 a_2 a_1 N_0 - y_2 y_1 a_2 \frac{X_1}{\phi} \right. \qquad (B-14)$$

$$\left. - y_2 \frac{X_2}{\phi} + y_2 N_3 \right]$$

B-6

For period 3, equation (B-8b) is

$$N_3 = h_3 a_3 N_2 - h_3 \frac{X_3}{\phi} + h_3 N_4 \qquad \text{(B-15)}$$

As before, since N_2 is known from equation (B-12c), we solve for N_3 in (B-15) as a function of N_0, $\frac{X_1}{\phi}$, $\frac{X_2}{\phi}$, $\frac{X_3}{\phi}$ and N_4 and substitute this expression into (B-14). Continuing in this way, the final expression for N_1 may be written as

$$
\begin{aligned}
N_1 = \; & y_1 a_1 N_0 - y_1 \frac{X_1}{\phi} + y_1 \bigg[y_2 y_1 a_2 a_1 N_0 - y_2 y_1 a_2 \frac{X_1}{\phi} - y_2 \frac{X_2}{\phi} \\
& + y_2 \bigg[y_3 y_2 y_1 a_3 a_2 a_1 N_0 - y_3 y_2 y_1 a_3 a_2 \frac{X_1}{\phi} \qquad \text{(B-14a)} \\
& - y_3 y_2 a_3 \frac{X_2}{\phi} - y_3 \frac{X_3}{\phi} + y_3 \bigg[y_4 \cdots y_1 a_4 \cdots a_1 N_0 \\
& - y_4 \cdots y_1 a_4 \cdots a_2 \frac{X_1}{\phi} - y_4 \cdots y_2 a_4 a_3 \frac{X_2}{\phi} \\
& - y_4 y_3 a_4 \frac{X_3}{\phi} - y_4 \frac{X_4}{\phi} + y_4 \bigg[\cdots \qquad \cdots \bigg]\bigg]\bigg]\bigg]
\end{aligned}
$$

This equation relates the demand for labor in period 1 (the present period) as a function of N_0, $\frac{X_1}{\phi}$, $\frac{X_2}{\phi}$,...., $\frac{X_k}{\phi}$,...., $\frac{X_{T-1}}{\phi}$ (the values of the exogenous variables in the present and future time periods) and N_T (labor used in period T, the end of the firm). Equation (B-14a) is a rather complicated expression and one that is not in readily useable form. There are certain properties that, intuitively, the coefficients on the X_ks should have. These include being less than one and have declining effect as k increases (i.e., the more distant into the future is the variable the less should be its effect on the firm's demand for labor in the present period).

In order to study the coefficients on the r.h.s. variables, it is useful to rearrange them and obtain:

for N_0:

$$y_1 a_1 + y_1^2 y_2 a_1 a_2 + y_1^2 y_2^2 y_3 a_1 a_2 a_3 + y_1^2 y_2^2 y_3^2 y_4 a_1 \cdots a_4$$

$$+ \ldots + y_1^2 \cdots y_t^2 y_{t+1} a_1 \cdots a_{t+1} + \ldots$$

$-\dfrac{x_1}{\phi}$:

$$y_1 + y_1^2 y_2 a_2 + y_1^2 y_2^2 y_3 a_2 a_3 + y_1^2 y_2^2 y_3^2 y_4 a_2 a_3 a_4$$

$$+ \ldots + y_1^2 \cdots y_t^2 y_{t+1} a_2 \cdots a_{t+1} + \ldots$$

$-\dfrac{x_2}{\phi}$:

$$y_1 y_2 + y_1 y_2^2 y_3 a_3 + y_1 y_2^2 y_3^2 y_4 a_3 a_4$$

$$+ \ldots + y_1 y_2^2 \cdots y_t^2 y_{t+1} a_3 \cdots a_{t+1} + \ldots$$

$-\dfrac{x_3}{\phi}$:

$$y_1 y_2 y_3 + y_1 y_2 y_3^2 y_4 a_4 + y_1 y_2 y_3^2 y_4^2 y_5 a_4 a_5$$

$$+ \ldots + y_1 y_2 y_3^2 \cdots y_t^2 y_{t+1} a_4 \cdots a_{t+1} + \ldots$$

.
.
.

$-\dfrac{x_k}{\phi}$:

$$y_1 \cdots y_k + y_1 \cdots y_{k-1} y_k^2 y_{k+1} a_{k+1}$$

$$+ \ldots + y_1 \cdots y_k^2 \cdots y_t^2 y_{t+1} a_{k+1} \cdots a_{t+1} + \ldots$$

.
.
.

N_T:

$$y_1 y_2 \cdots y_T = \prod_{i=1}^{T} y_i .$$

We now make a simplifying assumption in order to make the problem tractable. This simplification involves equations (B-9) and (B-10). Specifically, it is assumed that these functions are constant over time, or

$$h = h_t = \frac{1}{2+r+\frac{cq}{\phi}} \ , \ \text{for all } t = 0,1,\dots \tag{B-9a}$$

$$a = a_t = 1+r \ , \ \text{for all } t = 0,1,\dots \tag{B-10a}$$

This assumption says that the price of capital q_t and the interest rate r_t are the same for each time period. Earlier, it was stated that W_t/ϕ and W_t/q_t may be allowed to grow at the same rate since this generalization is also sufficient to provide a tractable solution. (Note that we are _not_ assuming that output is expected to remain constant.)

Either assumption enables us to work more easily with the y function already introduced (equation (B-13)). Note that this equation is a fairly complicated continued fraction. With a_t and h_t constant, it would look as follows:

$$y_t = \cfrac{h}{1-\cfrac{hah}{1-\cfrac{hah}{1-\underline{hah}}}} \qquad \text{t-terms}$$

The variable of interest is the product of the y_i's since that is what has been derived in equation (B-14a). In order to show the pattern of y_i, start with y_1, $y_1 y_2$, $y_1 y_2 y_3$,....,

or

$$y_1 = h$$

$$y_1 y_2 = h \frac{h}{1-h^2 a} = \frac{h^2}{1-h^2 a}$$

$$y_1 y_2 y_3 = h \frac{h}{1-h^2 a} \frac{h}{1- \frac{h^2 a}{1-h^2 a}} = \frac{h^3}{1-2h^2 a}$$

$$y_1 y_2 y_3 y_4 = \frac{h^4}{1-3h^2 a + \left(h^2 a\right)^2}$$

$$y_1 y_2 y_3 y_4 y_5 = \frac{h^5}{1-4h^2 a + 3\left(h^2 a\right)^2}$$

and so on.

Let D_t be equal to the denominator of the product $y_1, \ldots, y_t$. Then the product of the y_i's may be written as

$$\prod_{i=1}^{s} y_i = \frac{h^s}{D_s} \qquad s \geq 1 \qquad (B-16)$$

Furthermore, there are two (equivalent) ways of characterizing D_t:

$$D_t = \sum_{i=0}^{(t-1)/2} \binom{t-i}{i} \left(-h^2 a\right)^i, \ t \text{ odd} \qquad (B-17)$$

$$\sum_{i=0}^{t/2} \binom{t-i}{i} \left(-h^2 a\right)^i, \ t \text{ even}$$

and

$$D_t = D_{t-1} - h^2 a D_{t-2} . \qquad (B-18)$$

Although equation (B-17) looks something like the binomial
expansion, it is not a particularly useful characterization
of the process. Equation (B-18) is, on the other hand, a
fairly simple second order difference equation and its
solution will turn out to be quite useful later on in the
paper.

First, as a means of solving the second order difference
equation given in equation (B-18), note that an equivalent
expression for the equation is given by

$$D_{t+2} - D_{t+1} + h^2 a D_t = 0. \tag{B-19}$$

This is a second order homogeneous difference equation with a
general solution of the following form:

$$D_t = c_1 d_1^t + c_2 d_2^t . \tag{B-20}$$

In order to solve (B-19), it is necessary to determine the
roots, d_1 and d_2, of the characteristic equation,

$$d^2 - d + h^2 a = 0$$

which has a solution

$$d = \frac{1 \pm \sqrt{1 - 4h^2 a}}{2} .$$

Then let

$$d_1 = \frac{1 + \sqrt{1 - 4h^2 a}}{2}, \quad d_2 = \frac{1 - \sqrt{1 - 4h^2 a}}{2} . \tag{B-21}$$

B-11

We shall use the facts that $D_0 = D_1 = 1$ to solve for the constants c_1 and c_2:[1]

$$D_0 = 1 = c_1 + c_2 \qquad\qquad (B-22)$$

or
$$c_2 = 1 - c_1$$

and

$$D_1 = 1 = c_1 d_1 + c_2 d_2$$

$$1 = c_1 \frac{1+\sqrt{1-4h^2 a}}{2} + \left(1-c_1\right) \frac{1-\sqrt{1-4h^2 a}}{2} .$$

Let $Y = \sqrt{1-4h^2 a}$.

We then have,

$$1 = c_1 \frac{1+Y}{2} + \left(1-c_1\right) \frac{1-Y}{2}$$

$$= \frac{c_1}{2} + \frac{c_1 Y}{2} + \frac{1-Y}{2} - \frac{c_1}{2} + \frac{c_1 Y}{2}$$

$$1+Y = 2c_1 Y, \text{ or}$$

$$\frac{1+Y}{2Y} = c_1$$

and from (B-22)

$$c_2 = 1 - \frac{1+Y}{2Y}$$

$$= \frac{Y-1}{2Y} .$$

---

[1] D_1 is obviously 1 since $y_1 = h_1$. To show $D_0 = 1$, use the relationship $D_2 = D_1 - h^2 a D_0$. Since $D_2 = 1 - h^2 a$ and $D_1 = 1$, then D_0 must equal 1.

Then using (B-20) and substituting in the value for Y,

$$D_t = c_1 d_1^t + c_2 d_2^t$$

$$= \left(\frac{1 + \sqrt{1 - 4h^2 a}}{2\sqrt{1 - 4h^2 a}} \right) \left(\frac{1 + \sqrt{1 - 4h^2 a}}{2} \right)^t$$

$$+ \left(\frac{-1 + \sqrt{1 - 4h^2 a}}{2\sqrt{1 - 4h^2 a}} \right) \left(\frac{1 - \sqrt{1 - 4h^2 a}}{2} \right)^t$$

(B-23)

or

$$D_t = \frac{1}{\sqrt{1 - 4h^2 a}} \left(\frac{1 + \sqrt{1 - 4h^2 a}}{2} \right)^{t+1}$$

$$- \frac{1}{\sqrt{1 - 4h^2 a}} \left(\frac{1 - \sqrt{1 - 4h^2 a}}{2} \right)^{t+1} .$$

Before using equation (B-23), it is necessary to reduce the solution to the coefficients on N_0, $\frac{X_1}{\phi}$, $\frac{X_2}{\phi}$, ... from a finite sum to a more useable expression. Fortunately, the task is not quite so formidable as it first appears. We begin the solution by summing the first two terms. Then the next term is added and so on. A definite pattern becomes evident and a proof by induction is used to show the relationship holds for all $t = 1, 2, \ldots, T$.

It turns out that this method of proof may be used to solve for all the coefficients. This section shows this explicitly for the coefficient on N_0, and for any $\frac{X_i}{\phi}$ where i<T.

To solve for the coefficient on N_0, sum the first two terms on N_0 and use the expression given by (B-16) to eliminate the product of the y_i's:[1]

$$ha + \frac{h^3 a^2}{D_1 D_2} = ha\left(1 + \frac{h^2 a}{D_1 D_2}\right)$$

$$= ha\left(\frac{D_1 D_2 + h^2 a}{D_1 D_2}\right)$$

It has been stated that $D_1 = 1$ and $D_2 = 1 - h^2 a$ so that

$$ha + \frac{h^3 a^2}{D_1 D_2} = ha\ \frac{1}{D_1 D_2} = \frac{ha D_1}{D_2}$$

Adding the next term, $\dfrac{h^5 a^3}{D_2 D_3}$

$$\frac{ha D_1}{D_1} + \frac{h^5 a^3}{D_2 D_3} = ha\ \frac{D_1 D_3 + \left(h^2 a\right)^2}{D_2 D_3}$$

Since $D_1 = 1$ and $D_3 = 1 - 2h^2 a$

$$ha\ \frac{D_1 D_3 + \left(h^2 a\right)^2}{D_2 D_3} = ha\ \frac{\left(1 - 2h^2 a + \left(h^2 a\right) 2\right)}{D_2 D_3}$$

But $1 - 2h^2 a + \left(h^2 a\right)^2 = \left(1 - h^2 a\right)^2 = D_2^2$

and so

$$\frac{ha D_1}{D_2} + \frac{h^5 a^3}{D_2 D_3} = \frac{ha D_2^2}{D_2 D_3} = \frac{ha D_2}{D_3}$$

---

[1]The terms on N_0 are given on page B-8 of the appendix.

We want to prove by induction that this condition holds for any t, that is,

$$\frac{haD_{t-1}}{D_t} + \frac{ha\left(h^2a\right)^t}{D_tD_{t+1}} = \frac{haD_t}{D_{t+1}} \qquad (B-24)$$

Assume (B-24) is true for some t. Rearranging it gives

$$ha\left(D_{t-1}D_{t+1} + \left(h^2a\right)^t\right) = haD_t^2$$

$$\left(h^2a\right)^t = D_t^2 - D_{t-1}D_{t+1} \qquad (B-25)$$

From (B-25)

$$\left(h^2a\right)^{t+1} = \left(h^2a\right)^t \ h^2a \qquad (B-26)$$

$$= \left(D_t^2 - D_{t-1}D_{t+1}\right)h^2a$$

We also know from (B-18) that $D_t = D_{t-1} - h^2aD_{t-2}$ for any t, so that equivalently,

$$D_t h^2a = D_{t+1} - D_{t+2} \qquad (B-27)$$

and

$$-h^2aD_{t-1} = D_{t+1} - D_t \qquad (B-27a)$$

So, using (B-25), (B-26), (B-27) and (B-27a),

$$\left(h^2a\right)^{t+1} = D_t\left(D_{t+1} - D_{t+2}\right) + D_{t+1}\left(D_{t+1} - D_t\right)$$

$$= D_tD_{t+1} - D_tD_{t+2} + D_{t+1}^2 - D_{t+1}D_t$$

$$= D_{t+1}^2 - D_tD_{t+2}$$

Thus (B-25), and by rearrangement, (B-24) holds with t replaced by t+1. This implies that the r.h.s. of (B-24) is the coefficient of N_0 when t+1 and t are replaced by T and T-1, respectively. Explicitly, this coefficient is

$$\delta_0 = \frac{haD_{T-1}}{D_T} \quad .$$

The solution for the other coefficients proceed similarly. For $-\frac{X_1}{\phi}$, the coefficient is equal to the coefficient on N_0 divided by a, or specifically

$$\delta_1 = \frac{hD_{T-1}}{D_T} \quad .$$

Since the remaining coefficients, γ_2, γ_3, γ_4, ... follow the same pattern, we need only a general proof, similar to the preceding one to show what they are. Therefore, in general, for any i, where $2 \le i < t < T$, we assume the following equation (B-28) holds for some t

$$\frac{h^i D_{t-1}}{D_t} + \frac{h^i D_{i-1}\left(h^2 a\right)^{t-i+1}}{D_t D_{t+1}} = \frac{h^i D_{t-i+1}}{D_{t+1}} \quad . \tag{B-28}$$

Rearranging, we have

$$h^i \left(D_{t-i}D_{t+1} + D_{i-1}\left(h^2 a\right)^{t-i+1}\right) = h^i D_t D_{t-i+1}$$

or

$$D_{i-1}\left(h^2 a\right)^{t-i+1} = D_t D_{t-i+1} - D_{t-i}D_{t+1} \quad . \tag{B-28a}$$

Using (B-28a) but substituting t+1 for t,

$$D_{i-1}\left(h^2 a\right)^{t-i+2} = D_{i-1}\left(h^2 a\right)^{t-i+1}h^2 a \tag{B-29}$$

$$= \left(D_t D_{t-i+1} - D_{t-i}D_{t+1}\right)h^2 a$$

But, once again, we use $D_t = D_{t-1} - h^2 a D_{t-2}$, which holds for any t, so

$$h^2 a D_t = D_{t+1} - D_{t+2}$$

and

$$-h^2 a D_{t-i} = D_{t-i+2} - D_{t-i+1} .$$

Substituting these expressions into (B-29) yields

$$D_{i-1}\left(h^2 a\right)^{t-i+2} = D_{t-i+1}\left(D_{t+1} - D_{t+2}\right) + D_{t+1}\left(D_{t-i+2} - D_{t-i+1}\right)$$
$$= D_{t-i+1} D_{t+1} - D_{t-i+1} D_{t+2} + D_{t+1} D_{t-i+2}$$
$$- D_{t+1} D_{t-i+1} = D_{t+1} D_{t-i+2} - D_{t-i+1} D_{t+2} .$$

Therefore (B-29) and hence (B-28) holds with t replaced by t+1. The coefficient of $-\dfrac{X_i}{\phi}$ is the r.h.s. of (B-28) with t = T-1.

The last coefficient is the one on N_T, labor in the final period. It has already been stated to be

$$\gamma_T = \prod_{i=1}^{T} y_i = \frac{h^T}{D_T} .$$

Collecting all of these terms gives an expression for labor demand in the present period as a function of last period's labor, the present and future values of the exogenous variables up to period T-1 and labor demand in the last period, T. This expression is given by:

$$N_1 = \frac{haD_{T-1}}{D_T} N_0 - \frac{hD_{T-1}}{D_T} \frac{X_1}{\phi} - \frac{h^2 D_{T-2}}{D_T} \frac{X_2}{\phi} - \dots - \frac{h^T}{D_T} N_T \; .$$

$$(B-30)$$

THE SOLUTION FOR LABOR DEMAND IN THE LIMIT

The coefficients in equation (B-29) are all functions of time, since D_t is a time dependent variable. In particular, for each coefficient two different time periods are important--namely, k, and T, the latter denoting the end of the firm's planning period. Intuitively, it would seem that these coefficients should not depend upon T. It does turn out that the coefficient on labor demanded far into the future (where "far into the future" means as $T \to \infty$) approaches O and the coefficients on the exogenous variables k periods in the future only depend upon the kth time period and no other. This section will derive this explicitly.

We begin by showing that the coefficient on N_T does go to zero as T. From equation (B-16),

$$\lim_{T \to \infty} \prod_{i=1}^{T} y_i = \lim_{T \to \infty} \frac{h^T}{D_T} \; .$$

Dividing this on top and bottom by d_1^T, and using equation (B-20), the solution for D_T, gives

$$\lim_{T \to \infty} \frac{\left(\frac{h}{d_1}\right)^T}{c_1 + c_2 \left(\frac{d_2}{d_1}\right)^T} \; .$$

By the definition of h, d_1, and d_2, it follows that h/d_1, and d_2/d_1 are each less than 1. Therefore, this limit, which is the coefficient of N_T as $T \to \infty$, is 0.

In a similar fashion are found the values of the coefficients on $-\dfrac{X_1}{\phi}$, $-\dfrac{X_2}{\phi}$, $\ldots -\dfrac{X_k}{\phi}$. For example, for the coefficient on N_0 and $-\dfrac{X_1}{\phi}$,

$$
\begin{aligned}
\lim_{T\to\infty} \frac{D_{T-1}}{D_T} &= \lim_{T\to\infty} \frac{c_1 d_1^{T-1} + c_2 d_2^{T-1}}{c_1 d_1^{T} + c_2 d_2^{T}} \\[2em]
&= \lim_{T\to\infty} \frac{c_1 - c_2 \left(\dfrac{d_2}{d_1}\right)^{T-1}}{c_1 d_1 - c_2 d_2 \left(\dfrac{d_2}{d_1}\right)^{T-1}} \\[2em]
&= \frac{1}{d_1} \\[1.5em]
&= \frac{1}{\dfrac{1 + \sqrt{1-4h^2 a}}{2}}
\end{aligned}
$$

Thus, the coefficients on N_0 and $-\dfrac{X_1}{\phi}$, are

$$
\delta_0 = \frac{2ha}{1 + \sqrt{1-4h^2 a}} \tag{B-31}
$$

and

$$\delta_1 = \frac{2h}{1 + \sqrt{1 - 4h^2 a}} \quad .$$

Analogously, it may be shown that for the coefficients δ_2, ..., δ_k the solutions are:

$$\delta_2 = \left(\frac{2h}{1 + \sqrt{1 - 4h^2 a}} \right)^2$$

.
.
.

$$\delta_k = \left(\frac{2h}{1 + \sqrt{1 - 4h^2 a}} \right)^k \quad .$$

The final form of the equation may then be written as

$$N_1 = \frac{2ha}{1 + \sqrt{1 - 4h^2 a}} \; N_0 - \sum_{k=1}^{\infty} \left(\frac{2h}{1 + \sqrt{1 - 4h^2 a}} \right)^k \frac{X_k}{\phi} \qquad \text{(B-32)}$$

or, after substituting in for the exogenous variables,

$$N_1 = \frac{2ha}{1 + \sqrt{1 - 4h^2 a}} \; N_0 - \sum_{k=1}^{\infty} \left(\frac{2h}{1 + \sqrt{1 - 4h^2 a}} \right)^k$$

$$\cdot \; \frac{1}{\phi} \; \cdot \; b_1 - W_k + d_1 Q_k$$

where $b_1 = bq$

and $\quad d_1 = -dq.$

These coefficients can be shown to be less than one. An interesting question, and one which serves as a check on the coefficients, is what happens as ϕ, the adjustment cost

parameter, gets larger and larger, eventually going to in-
finite cost. It might be expected that as the cost of
changing the labor force becomes inordinately expensive, it
will then stay constant at N_o. To show this, remember the
definition of h, which is

$$h = \frac{1}{2+r+\frac{cq}{\phi}} \quad .$$

As $\phi \to \infty$, $h = \frac{1}{2+r}$, which, in turn, leads to the following:

$$\frac{2ha}{1+\sqrt{1-4h^2a}} = \frac{2(1+r)\frac{1}{(2+r)}}{1+\sqrt{1-\frac{4(1+r)}{(2+r)^2}}} = \frac{\frac{2+2r}{2+r}}{1+\sqrt{\frac{r^2}{(2+r)^2}}}$$

$$= \frac{\frac{2+2r}{2+r}}{\frac{2+2r}{2+r}} = 1.$$

Also, since $\frac{1}{\phi} \to 0$ as $\phi \to \infty$, the coefficients on the X_k terms
all go to zero. This means that $N_1 = N_o$.

APPENDIX C

THE REGRESSION EQUATIONS FOR THE OUTPUT MODEL

APPENDIX C

THE REGRESSION EQUATIONS FOR THE OUTPUT MODEL

To estimate the labor demand equation given by equation (13), we had to form series on expected output by industry. This section presents the results of estimating the three-equation system described by equations (14), (16), and (17) using ordinary least squares (with a correction for autocorrelation of the residuals when necessary). Following the presentation of the regression estimates, a brief discussion of the results is provided.

GENERATING ESTIMATES OF EXPECTED OUTPUT

To generate the (rational) forecasts described earlier, equations are estimated for total demand, imports, relative prices, and GNP. Forecasts of domestic demand are then computed as the difference between the forecasts of total demand and imports (equation (12)). The actual regression equations which are used to generate these forecasts are presented on pages C-4 to C-15.

The first equation for each industry describes the determinants of the total demand for output of that industry. Total demand depends upon real GNP (a scale variable), lags in GNP (which are included to capture cyclical factors), the relative price of the industry's products, a time trend, and dummy variables to correct for seasonal factors.[1]

---

[1] In addition, a number of dummy variables were used to capture particular events affecting an industry's output, imports, or price. For example, imports in the paper industry were greatly affected by a Canadian paper strike in the fourth quarter of 1975. As expected, the coefficient on the dummy variable for this is negatively related to industry imports and quite significantly so. For forecasting purposes, it is important to include these dummy variables, since their exclusion is likely to result in a poorly fitted equation and poor forecasts.

C-1

This general form seems to have worked well. In every industry but two (26 and 31), the level of GNP has a significantly positive effect. Cyclicality, represented by the change in GNP over one or four periods or by various lags in GNP, also has strong positive effects in most industries.

Since the logarithmic form is used, the coefficients give the elasticities directly. For example, as GNP increases by one percent, total demand for the products of industry 22 increases by approximately 1.14 percent. In most cases, elasticities are near unity (although industry 37 exhibits a far higher elasticity). Similarly, the elasticity of demand with respect to cyclical factors is quite plausible in every industry and, on average, is also close to one.

Another influence on industry demand is the industry's price relative to a price of all manufactured goods. The response to this price is a measure of substitution between the products in one industry and all other manufacturing industries. In every industry but one (industry 22), the price measure exhibits a significantly negative effect on industry demand. For most industries, the simple relative price term works well. In four industries (26, 30, 34, and 35), an average of relative prices over four periods is used and in one industry (33), an average over eight periods is used. This averaging represents a price change that takes several quarters to take effect. The elasticities also are plausible in magnitude, most being less than or equal to one.

Since imports and relative prices are exogenous variables in the model, the general form for each was kept as simple as possible. Both sets of equations were estimated using a distributed lag model (up to two lags), with a time trend, and seasonal dummy variables included. All price equations

had R^2 above .80 and most were over .90. The import equations generally had somewhat more unexplained variance, although most had R^2 above .85.

The last forecasting equation to be estimated is the GNP equation. It, too, was estimated as a two quarter distributed lag with a time trend and seasonal dummy variables included. The specification worked well, and the equation has an R^2 of .986.

INDUSTRY 22 - TEXTILE MILL PRODUCTS

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = \underset{(-1.14)}{-3.817} + \underset{(3.41)}{1.142} \ln(RGNP) + \underset{(1.93)}{0.695} \ln \frac{RGNP}{RGNP(-4)}$$

$$\underset{(-1.34)}{- 0.005} TIME \underset{(-0.06)}{-0.004} D1 + \underset{(1.07)}{0.0083} D2 - \underset{(-8.57)}{0.0599} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.838
DW : 1.264
ρ : 0.863

2. Real Imports

$$\ln \frac{M}{WPI} = \underset{(2.67)}{0.346} + \underset{(6.23)}{0.705} \ln \frac{M(-1)}{WPI(-1)} - \underset{(-1.54)}{0.0026} TIME$$

Range : Q1 1969 to Q4 1977
R^2 : 0.621
DW : 2.06

3. Relative Price

$$\ln \frac{WPI}{GWPI} = \underset{(1.79)}{0.017} + \underset{(12.02)}{1.458} \ln \frac{WPI(-1)}{GWPI(-1)} - \underset{(-5.87)}{0.708} \ln \frac{WPI(-2)}{GWPI(-2)}$$

$$\underset{(-3.11)}{- 0.001} TIME - \underset{(-1.64)}{0.012} D1 + \underset{(0.33)}{0.002} D2 - \underset{(-0.93)}{0.007} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.958
DW : 2.09

NOTE: t-statistics are listed in parentheses below the coefficients.

INDUSTRY 23 - APPAREL AND OTHER TEXTILE PRODUCTS

1. Total Output

$$\ln \frac{SHIP + INV + M}{WPI} = -2.178 + 0.931 \ln(RGNP) + 0.924 \ln \frac{RGNP}{RGNP(-4)}$$
$$(-0.61) \quad (2.34) \qquad\qquad (1.80)$$

$$- 0.856 \ln \frac{WPI}{GWPI} - 0.019 \; TIME + 0.011 \; D1$$
$$(-2.77) \qquad\qquad (-3.50) \qquad (1.40)$$

$$+ 0.032 \; D2 + 0.033 \; D3$$
$$(3.64) \qquad (4.13)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.717
DW : 2.06
ρ : 0.865

2. Real Imports

$$\ln \frac{M}{WPI} = 3.152 + 0.565 \ln \frac{M(-1)}{WPI(-1)} + 0.014 \; TIME - 0.003 \; D1$$
$$(2.83) \quad (3.84) \qquad\qquad (2.79) \qquad\quad (-0.08)$$

$$+ 0.119 \; D2 + 0.318 \; D3$$
$$(2.13) \qquad (6.50)$$

Range : Q1 1969 to Q4 1977
R^2 : 0.966
DW : 1.93

3. Relative Price

$$\ln \frac{WPI}{GWPI} = 0.020 + 1.508 \ln \frac{WPI(-1)}{GWPI(-1)} - 0.582 \ln \frac{WPI(-2)}{GWPI(-2)}$$
$$(2.24) \; (10.57) \qquad\qquad (-4.29)$$

$$- 0.008 \; TIME - 0.019 \; D1 - 0.009 \; D2 - 0.007 \; D3$$
$$(-2.02) \qquad (-2.1) \qquad (-1.60) \qquad (-0.84)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.995
DW : 2.12
     : -0.45

INDUSTRY 26 - PAPER AND ALLIED PRODUCTS

1. Total Output

$$\ln \frac{SHIP + INV + M}{WPI} = \underset{(-2.07)}{-5.001} + \underset{(3.72)}{1.308} \ln \frac{RGNP}{RGNP(-1)} - \underset{(-2.63)}{1.059} \ln(AP4)$$

$$- \underset{(-1.25)}{0.003} \; TIME + \underset{(2.40)}{0.019} \; D1 + \underset{(4.31)}{0.035} \; D2 - \underset{(-0.57)}{0.004} \; D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.811
DW : 1.593
ρ : 0.625

2. Real Imports

$$\ln \frac{M}{WPI} = \underset{(5.45)}{0.773} + \underset{(2.27)}{0.693} \ln \frac{M(-1)}{WPI(-1)} - \underset{(-2.53)}{0.329} \ln \frac{M(-2)}{WPI(-2)}$$

$$+ \underset{(4.60)}{0.006} \; TIME - \underset{(-2.86)}{0.085} \; D1 - \underset{(-0.72)}{0.020} \; D2 - \underset{(-4.35)}{0.132} \; D3$$

$$- \underset{(-4.95)}{0.300} \; D5*$$

Range : Q1 1969 to Q4 1977
R^2 : 0.852
DW : 1.725

3. Relative Price

$$\ln \frac{WPI}{GWPI} = \underset{(3.46)}{.011} + \underset{(7.68)}{.646} \ln \frac{WPI(-1)}{GWPI(-1)} - \underset{(-3.90)}{.03} \; DUM73**$$

Range : Q1 1968 to Q4 1977
R^2 : .83
DW : 2.30

* Dummy variable used to represent a Canadian paper strike; D5 = 1 in Q4 1975, 0 elsewhere.

** Dummy variable used to account for large drop in relative price of paper, it may reflect the effects of price controls; DUM73 = 1 in Q1 to Q4 1973, 0 elsewhere.

INDUSTRY 30 - RUBBER AND PLASTICS PRODUCTS

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = -3.926 + 1.086 \ln(RGNP) + 0.918 \ln\frac{RGNP}{RGNP(-4)}$$
$$(-1.12) \quad (2.16) \quad\quad\quad (1.77)$$

$$- 0.971 \ln(AP4) + 0.014\ D1 + 0.050\ D2$$
$$(-2.01) \quad\quad\quad (1.19) \quad\quad (3.94)$$

$$- 0.019\ D3$$
$$(-1.68)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.776
DW : 1.856
ρ : 0.738

2. Real Imports

$$\ln \frac{M}{WPI} = -0.117 + 0.732 \ln \frac{M(-1)}{WPI(-1)} + 0.007\ TIME + 0.201\ D1$$
$$(-1.67) \quad (5.76) \quad\quad\quad\quad (1.69) \quad\quad (3.72)$$

$$+ 0.181\ D2 + 0.119\ D3$$
$$(3.53) \quad\quad (2.33)$$

Range : Q1 1969 to Q4 1977
R^2 : 0.920
DW : 1.88

3. Related Price

$$\ln \frac{WPI}{GWPI} = 0.011 + 1.220 \ln \frac{WPI(-1)}{GWPI(-1)} - 0.356 \ln \frac{WPI(-2)}{GWPI(-2)}$$
$$(1.00) \quad (7.60) \quad\quad\quad\quad (-2.20)$$

$$- 0.0007\ TIME - 0.021\ D1 - 0.002\ D2 - 0.0008\ D3$$
$$(-1.39) \quad\quad (-2.67) \quad\quad (-0.30) \quad\quad (-0.10)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.955
DW : 2.25

INDUSTRY 31 - LEATHER AND LEATHER PRODUCTS

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = \underset{(43.01)}{2.718} + \underset{(2.29)}{1.294} \ln \frac{RGNP}{RGNP(-4)} - \underset{(-2.20)}{0.649} \ln \frac{WPI}{GWPI}$$

$$- \underset{(-6.66)}{0.015} TIME + \underset{(3.43)}{0.052} D1 + \underset{(2.23)}{0.038} D2 + \underset{(0.39)}{0.006} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.684
DW   : 2.16
ρ : 0.628

2. Real Imports

$$\ln \frac{M}{WPI} = \underset{(1.40)}{0.628} + \underset{(2.90)}{0.476} \ln \frac{M(-1)}{WPI(-1)} + \underset{(2.71)}{0.440} \ln \frac{M(-2)}{WPI(-2)}$$

$$+ \underset{(1.26)}{0.058} D1 + \underset{(0.37)}{0.015} D2 + \underset{(1.60)}{0.074} D3$$

Range : Q1 1969 to Q4 1977
R^2 : 0.845
DW   : 2.17

3. Relative Price

$$\ln \frac{WPI}{GWPI} = \underset{(1.60)}{0.019} + \underset{(10.86)}{1.485} \ln \frac{WPI(-1)}{GWPI(-1)} - \underset{(4.53)}{0.612} \ln \frac{WPI(-2)}{GWPI(-2)}$$

$$- \underset{(-1.54)}{0.0006} TIME - \underset{(-1.43)}{0.014} D1 + \underset{(0.40)}{0.004} D2 - \underset{(-2.01)}{0.020} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.939
DW   : 2.19

INDUSTRY 32 - STONE, CLAY, AND GLASS PRODUCTS

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = \begin{matrix}-10.553 \\ (-3.47)\end{matrix} + \begin{matrix}0.877 \\ (2.36)\end{matrix} \ln(RGNP) + \begin{matrix}1.099 \\ (2.78)\end{matrix} \ln \frac{RGNP}{RGNP(-1)}$$

$$- \begin{matrix}0.519 \\ (-1.89)\end{matrix} \ln \frac{WPI}{GWPI} - \begin{matrix}0.044 \\ (-5.99)\end{matrix} D1 + \begin{matrix}0.041 \\ (5.16)\end{matrix} D2$$

$$+ \begin{matrix}0.025 \\ (3.55)\end{matrix} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.879
DW   : 2.01
ρ   : 0.977

2. Real Imports

$$\ln \frac{M}{WPI} = \begin{matrix}.020 \\ (.39)\end{matrix} + \begin{matrix}.837 \\ (8.66)\end{matrix} \ln \frac{M(-1)}{WPI(-1)} - \begin{matrix}.019 \\ (-.39)\end{matrix} D1 + \begin{matrix}.134 \\ (2.71)\end{matrix} D2$$

$$+ \begin{matrix}.072 \\ (1.50)\end{matrix} D3$$

Range : Q1 1969 to Q4 1969
R^2 : .73
DW   : 2.19

3. Relative Price

$$\ln \frac{WPI}{GWPI} = \begin{matrix}0.007 \\ (1.63)\end{matrix} + \begin{matrix}1.408 \\ (9.50)\end{matrix} \ln \frac{WPI(-1)}{GWPI(-1)} - \begin{matrix}0.526 \\ (-3.54)\end{matrix} \ln \frac{WPI(-2)}{GWPI(-2)}$$

$$- \begin{matrix}0.0008 \\ (-0.13)\end{matrix} D1 - \begin{matrix}0.006 \\ (-1.05)\end{matrix} D2 - \begin{matrix}0.013 \\ (-2.18)\end{matrix} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.884
DW   : 2.09

INDUSTRY 33 - PRIMARY METALS

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = -3.63 + 1.22 \ln(RGNP(-1)) - 2.53 \ln(AP8)$$
$$(-2.19) \quad (5.15) \qquad\qquad (-4.79)$$

$$+ .077\ D1 + .07\ D2 - .014\ D3 + .1\ D71^*$$
$$(1.22) \qquad (4.40) \quad (-1.03) \qquad (4.89)$$

Range : Q1 1968 to Q4 1977
R^2 : .791
DW   : 1.40
ρ : .59

2. Real Imports

$$\ln \frac{M}{WPI} = 1.34 + .452 \ln \frac{M(-1)}{WPI(-1)} - .118\ D1 + .04\ D2$$
$$(3.55)(2.89) \qquad\qquad (-1.93) \qquad (.625)$$

$$- .031\ D3 - .145\ DQ^{**}$$
$$(-.499) \quad (-2.52)$$

Range : Q1 1969 to Q4 1977
R^2 : .584
DW   : 1.73

3. Relative Price

$$\ln \frac{WPI}{GWPI} = -.005 + 1.34 \ln \frac{WPI(-1)}{GWPI(-1)} - .687 \ln \frac{WPI(-2)}{GWPI(-2)}$$
$$(.74) \quad (10.97) \qquad\qquad (-5.64)$$

$$+ .0009\ TIME$$
$$(3.05)$$

Range : Q1 1968 to Q4 1977
R^2 : .886
DW   : 1.927
ρ : .28

 * Dummy variable used to represent inventory adjustment
because of labor negotiations; D71 = 1 in Q1 and Q2 1971,-1 in
Q3 and Q4 1971, 0 elsewhere.

** Dummy variable used to represent quota on imports from 1969
to 1971; DQ = 1 in Q1 to Q4 for 1969, 1970, and 1971; 0
elsewhere.

INDUSTRY 34 – FABRICATED METAL PRODUCTS

1. Total Output

$$\ln \frac{SHIP+\ INV+M}{WPI} = -5.264 + 1.439 \ln(RGNP(-1))$$
$$\phantom{\ln \frac{SHIP+\ INV+M}{WPI} =} (-2.23)\quad (4.18)$$

$$+\ 0.921\ \ln\frac{RGNP}{RGNP(-4)}\ -\ 1.577\ \ln(AP4)$$
$$ (4.11)\qquad\qquad (-0.30)$$

$$-\ 0.0086\ TIME + 0.017\ D1 + 0.063\ D2 + 0.003\ D3$$
$$(-3.91)\qquad\quad (1.80)\qquad (6.30)\qquad (0.37)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.911
DW : 2.01
ρ : 0.219

2. Real Imports

$$\ln \frac{M}{WPI} = 0.065 + 0.686\ \ln \frac{M(-1)}{WPI(-1)} + 0.007\ TIME - 0.004\ D1$$
$$\phantom{\ln \frac{M}{WPI} =} (0.98)\quad (4.98)\qquad\qquad (1.78)\qquad (-0.08)$$

$$-\ 0.089\ D2 - 0.003\ D3$$
$$(-1.81)\qquad (-0.05)$$

Range : Q1 1969 to Q4 1977
R^2 : 0.877
DW : 2.03

3. Relative Price

$$\ln \frac{WPI}{GWPI} = 0.005 + 1.303\ \ln \frac{WPI(-1)}{GWPI(-1)} - 0.498\ \ln \frac{WPI(-2)}{GWPI(-2)}$$
$$\phantom{\ln \frac{WPI}{GWPI} =} (1.13)\quad (8.70)\qquad\qquad (-3.32)$$

$$-\ 0.009\ D1 - 0.0005\ D2 - 0.0006\ D3$$
$$(-1.49)\qquad (-0.07)\qquad (-0.09)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.816
DW : 2.34

INDUSTRY 35 – MACHINERY, EXCEPT ELECTRICAL

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = \underset{(-3.35)}{-11.513} + \underset{(1.83)}{0.778}\ \ln(RGNP) + \underset{(1.85)}{0.833}\ \ln(RGNP(-1))$$

$$+ \underset{(1.55)}{0.693}\ \ln(RGNP(-2)) - \underset{(-3.13)}{1.400}\ \ln(AP4)$$

$$+ \underset{(4.78)}{0.040}\ D1 + \underset{(5.88)}{0.051}\ D2 - \underset{(-6.67)}{0.051}\ D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.888
DW : 0.99
ρ : 0.963

2. Real Imports

$$\ln \frac{M}{WPI} = \underset{(1.74)}{0.384} + \underset{(1.97)}{0.346}\ \ln \frac{M(-1)}{WPI(-1)} + \underset{(1.88)}{0.332}\ \ln \frac{M(-2)}{WPI(-2)}$$

$$+ \underset{(1.63)}{0.007}\ TIME + \underset{(1.50)}{0.069}\ D1 - \underset{(-2.71)}{0.129}\ D2 - \underset{(-0.12)}{0.006}\ D3$$

Range : Q1 1969 to Q4 1977
R^2 : 0.90
DW : 1.83

3. Relative Price

$$\ln \frac{WPI}{GWPI} = \underset{(2.20)}{0.009} + \underset{(9.70)}{1.440}\ \ln \frac{WPI(-1)}{GWPI(-1)} - \underset{(-3.52)}{0.523}\ \ln \frac{WPI(-2)}{GWPI(-2)}$$

$$- \underset{(-2.71)}{0.016}\ D1 - \underset{(-1.56)}{0.009}\ D2 - \underset{(-2.01)}{0.011}\ D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.920
DW : 2.34

INDUSTRY 36 – ELECTRICAL EQUIPMENT AND SUPPLIES

1. Total Output

$$\ln \frac{SHIP + INV + M}{WPI} = -13.741 + 1.218 \ln(RGNP) + 0.628 \ln(RGNP(-1))$$
$$(-3.35) \quad (2.51) \qquad\qquad (1.08)$$

$$+ 0.756 \ln(RGNP(-2)) - 0.536 \ln \frac{WPI}{GWPI}$$
$$(1.52) \qquad\qquad\qquad (-1.32)$$

$$- 0.035 \ D1 - 0.002 \ D2 - 0.060 \ D3$$
$$(-3.38) \qquad (-0.20) \qquad (-6.01)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.78
DW : 1.48
ρ : 0.942

2. Real Imports

$$\ln \frac{M}{WPI} = 0.351 + 1.070 \ln \frac{M(-1)}{WPI(-1)} - 0.307 \ln \frac{M(-2)}{WPI(-2)}$$
$$(2.49) \quad (6.02) \qquad\qquad (-1.75)$$

$$+ 0.007 \ TIME - 0.155 \ D1 + 0.103 \ D2 - 0.008 \ D3$$
$$(1.95) \qquad\quad (-4.21) \qquad (2.14) \qquad (-0.20)$$

Range : Q1 1969 to Q4 1977
R^2 : 0.965
DW : 2.06

3. Relative Price

$$\ln \frac{WPI}{GWPI} = 0.014 + 1.398 \ln \frac{WPI(-1)}{GWPI(-1)} - 0.491 \ln \frac{WPI(-2)}{GWPI(-2)}$$
$$(1.51) \quad (9.02) \qquad\qquad\quad (-3.16)$$

$$- 0.0007 \ TIME - 0.016 \ D1 - 0.010 \ D2 - 0.009 \ D3$$
$$(-1.41) \qquad\quad (-2.81) \qquad (-1.75) \qquad (-1.66)$$

Range : Q1 1968 to Q4 1977
R^2 : 0.988
DW : 2.28

INDUSTRY 37 - TRANSPORTATION EQUIPMENT

1. Total Output

$$\ln \frac{SHIP+ INV+M}{WPI} = \begin{array}{c} -16.586 \\ (-4.49) \end{array} + \begin{array}{c} 3.177 \\ (5.92) \end{array} \ln(RGNP) + \begin{array}{c} 1.474 \\ (1.51) \end{array} \ln \frac{RGNP}{RGNP(-1)}$$

$$\begin{array}{c} - 0.551 \\ (-1.83) \end{array} \ln \frac{WPI}{GWPI} \begin{array}{c} - 0.015 \\ (-3.98) \end{array} TIME \begin{array}{c} - 0.017 \\ (-0.74) \end{array} D1$$

$$\begin{array}{c} + 0.030 \\ (1.27) \end{array} D2 \begin{array}{c} - 0.127 \\ (-5.41) \end{array} D3$$

Range : Q1 1963 to Q4 1977
R^2 : 0.831
DW   : 1.81
ρ : 0.361

2. Real Import

$$\ln \frac{M}{WPI} = \begin{array}{c} 0.942 \\ (2.51) \end{array} + \begin{array}{c} 0.346 \\ (2.32) \end{array} \ln \frac{M(-1)}{WPI(-1)} + \begin{array}{c} 0.302 \\ (2.03) \end{array} \ln \frac{M(-2)}{WPI(-2)}$$

$$\begin{array}{c} + 0.006 \\ (1.56) \end{array} TIME + \begin{array}{c} 0.056 \\ (1.07) \end{array} D1 + \begin{array}{c} 0.065 \\ (1.61) \end{array} D2 - \begin{array}{c} -0.186 \\ (-4.07) \end{array} D3$$

Range : Q1 1969 to Q4 1977
R^2 : 0.886
DW   : 1.61
ρ : 0.437

3. Relative Price

$$\ln \frac{WPI}{GWPI} = \begin{array}{c} 0.032 \\ (3.75) \end{array} + \begin{array}{c} 1.530 \\ (10.56) \end{array} \ln \frac{WPI(-1)}{GWPI(-1)} \begin{array}{c} - 0.613 \\ (-4.20) \end{array} \ln \frac{WPI(-2)}{GWPI(-2)}$$

$$\begin{array}{c} - 0.0004 \\ (-1.05) \end{array} TIME \begin{array}{c} - 0.053 \\ (-6.78) \end{array} D1 \begin{array}{c} - 0.032 \\ (-5.19) \end{array} D2 \begin{array}{c} - 0.035 \\ (-5.56) \end{array} D3$$

Range : Q1 1968 to Q4 1977
R^2 : 0.977
DW   : 2.15

REAL GROSS NATIONAL PRODUCT

$$\ln(RGNP) = 1.048 + 1.346 \ln(RGNP(-1)) - .5 \ln(RGNP(-2))$$
$$(2.09) \quad (3.84) \qquad\qquad (-3.25)$$

$$+ .001 \text{ TIME} + .011 \text{ D1} + .0035 \text{ D2} + .008 \text{ D3}$$
$$(2.17) \qquad\quad (2.42) \qquad (.784) \qquad (1.76)$$

Range : Q1 1968 to Q4 1977
R^2 : .986
DW   : 2.22

APPENDIX D

IMPRECISION ON THE MEASUREMENT OF b_4

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IMPRECISION ON THE MEASUREMENT OF b_4

The empirical estimates of the coefficient on the lagged
dependent variable have been found to vary over a greater
range than the theoretical model would suggest.[1] According
to the model, the value of this coefficient should be greater
than one since it is composed of the sum of 1 plus the
interest rate. For the eight industries where b_3 was found
to be significant (which therefore implies that expectations
of future output matter) the coefficient on last period's
labor was estimated to be greater than one in four industries
(23, 26, 32, 36), and it was found to be less than one in the
remaining four (22, 33, 34, 35).

This breakdown is somewhat artificial because a simple t-test
on this coefficient by industry shows that in almost every
case (industry 33 is the exception once autocorrelation is
taken account of), b_4 is not significantly different from 1.
This is true for values of b_4 both below and above 1. It
would appear, therefore that there is a great deal of random
error associated with estimating the value of 1 plus the real
interest rate.

The lack of precision of b_4, the estimate of 1+r, may be
illustrated through the construction of confidence intervals

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[1]This discussion will focus on estimates obtained from either
the original estimation or the estimates corrected for auto-
correlation. The exception to this will be for industry 35
where relative wage variables appear to be useful additional
explanatory variables and are necessary for the convergence
of the nonlinear routine.

for b_4. We assume that 1+r would be equal for all industries so that the b_4's should also be equal. We shall construct the interval around an average b_4 over the eight industries listed previously. To get an approximate standard error for the average of b_4 across industries, we proceed as if the coefficient had no correlation across industries and approximate the distribution of the average b_4 with the normal distribution.[1] using these assumptions, the standard error of b_4 is formed as:

$$\sigma\left(\overline{b}_4\right) = \frac{1}{n}\sqrt{\sum_{i=1}^{8} \sigma_i^2\left(b_4\right)}$$

where $\sigma\left(\overline{b}_4\right)$ is the standard error of b_4

σ_i^2 is the variance of b_4 for industry i.

The 95 percent confidence interval may then be formed as:

$$\overline{b}_4 \pm \sigma\left(\overline{b}_4\right) 1.96 \ .$$

It turns out that $\overline{b}_4$ = .86 and $\sigma\left(\overline{b}_4\right)$ = .126 so that the interval has .61 as a lower limit and 1.1 as an upper limit.

Thus, the existence of values of b_4 below 1 is explicable, in part, by pure sampling error. There still may remain some bias in b_4. The cause of the bias is discussed in the next section.

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[1] On a theoretical level, there is no need to assume independence between the b_4's across industries. A joint estimating procedure, such as the seemingly unrelated regression model, could be run with b_4 constrained to be equal across all industries. On a practical level, however, getting the estimates would be extremely difficult and so is not attempted here.

Other Possible Problems with b_4

There may be bias in b_4 due to the use of only eight forecasts in the nonlinear regression. If more distant forecasts were important as determinants of labor demand, their exclusion would lead to omitted variable bias. In particular, since the forecasts are all positively correlated, their omission would cause b_3 to be biased upward. Further, if the estimates of b_3 and b_4 are negatively correlated (and according to the theory, they are since $b_3 = \frac{\lambda_1}{a}$ and $b_4 = a$), then a higher value for b_3 would result in a lower value for b_4. In fact, the estimated covariances for b_3 and b_4 obtained from the nonlinear regressions (where b_3 is significant) confirm this negative correlation.

To determine whether this hypothesis was reasonable, two industries were chosen and further expectations of output were formed. The two industries were 22 (textiles) and 34 (fabricated metals). Expectations up to 16 periods in the future were formed and included in the labor demand regression.

For industry 34, which did not need an autocorrelation correction, the extra leads seemed to create negligible differences in estimated coefficients. Including expectations up to eight quarters in the future seems sufficient.

For industry 22, which did need an autocorrelation correction, the resulting estimates were somewhat different when 12 leads were included. (The equation when 16 leads were used was virtually the same as for 12.) The results for 12 leads are given below:

$$N_{t+1} = 244.04 + 2.24 \sum_{i=1}^{13} .779^i \ Q^*_{t+i} + (.779)(.736)N_t$$
$$\quad\ \ (.867)(5.09) \qquad (9.52) \qquad\qquad\qquad (2.89)$$

$$-2.21(t+1) \ -3.11 \ D1 + 6.69 \ D2 + 12.1 \ D3$$
$$(-1.58) \qquad (-.86) \qquad (1.64) \qquad (3.47)$$

$$\rho = \ .774$$
$$(4.58)$$

Comparing this regression with the one for industry 22 given in table 5, it is clear that the major differences have more to do with the precision of the estimates than the estimates themselves. The coefficient, b_2, is similar, going from 2.67 to 2.24, but its t-statistic moves from under 1 to over 5. The coefficient, b_3, is also close to the previous result (moving from .751 to .779), but its t-statistic is now 9.52. The coefficient of greatest interest in this section, b_4, goes from about .64 to about .74. Although in relative terms this is a fairly large increase, statistically the increase is insignificant. Nevertheless, the addition of more expectation variables does change the coefficient in a direction consistent with economic theory.

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